臺灣大學數學系
九十九學年度碩士班甄試試題
科目：數值分析(含程式設計) 2009.10.30

1. (25%)Hermite interpolation and Simpson rule:

   (a) Find a polynomial $p(x)$ of degree 3 that interpolates a function $f(x)$ at x_{-1}, x_0 and x_1 with $p'(x_0) = f'(x_0)$.

   (b) What is the error of the above interpolation?

   (c) Write down the Simpson's integration rule on an interval $[a, b]$ with equal partition $a = x_0, ..., x_n = b$ and show the error estimate.

2. (25%) Root-finding: Consider the function $f(x) = \tan^{-1} x - \frac{2x}{1+x^2}$. The problem is to find a positive root of f.

   (a) Derive a fixed-point method to find a positive root. Write down your pseudo-code.

   (b) What is the Newton's root-find method? (Write a pseudo-code) Can you find the positive root by Newton's method?

   (c) Show the quadratic convergence behavior of Newton's method.

3. (25%) Least-squares solutions: Consider an $m \times n$ matrix A and a vector $b \in \mathbb{R}^n$.

   (a) Show that the minimal solution x^* of

   $$H(x) = \frac{1}{2}\|Ax - b\|^2 := \frac{1}{2}\sum_i \left(\sum_j a_{i,j}x_j - b_i\right)^2$$

   satisfies the Euler-Lagrange equation:

   $$A^T(Ax - b) = 0.$$

   (b) Show that the gradient of H at a point x is $A^T(Ax - b)$?

   (c) Consider the descent gradient method:

   $$x^{k+1} = x^k - \lambda A^T(Ax^k - b).$$

   Under what condition on the parameter λ does the method converge? Prove your argument.

4. (25%) Choose any topics in numerical analysis or scientific computing that you like and give a brief description on it.