台灣大學數學系九十八學年度碩士班甄試入學考試試題
數值分析（含程式設計）

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1. (20%) Suppose we are working on a certain floating point system wih $A=\begin{bmatrix}\delta & 1\\ 1 & 1\end{bmatrix}$. Here δ is a positive number that is smaller than the unit round-off machine precision ε. (a) Perform standard LU factorization on A without any row or column permutation to obtain the unit lower triangular matrix L_1 and the upper triangular matrix U_1. (b) Is $A = L_1U_1$? If yes, explain why. If no, explain how you may compute the LU factorization to obtain L_2 and U_2, so that $A = L_2U_2$.

2. (20%) Perform two steps of Newton's method for solving the nonlinear system

$$f(x)=\begin{cases}x_1+2x_2=2,\\ x_1^2+4x_2^2=4.\end{cases}$$ Suppose the initial guess $\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}$.

3. (20%) Let $u\in\Re^n$ with $\|u\|_2=1$. The $n\times n$ matrix $P=I-2uu^T$ is called a Householder transformation. (a) Show that a Householder transformation is symmetric, orthogonal, and satisfies $P^{-1}=P$. (b) Explain how you may use Householder transformation P_1 such that

$$P_1\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}=\begin{bmatrix}\bar{x}_1\\ 0\\ \vdots\\ 0\end{bmatrix}.$$

4. (20%) Given a function f defined on $[a,b]$ and a set of nodes $a=x_0<x_1<x_2<\cdots<x_n=b$. (a) Define a natural cubic spline interpolant S for f. (b) Write down the resulting linear system for determining the coefficients of the piecewise cubic polynomials and then show that the natural cubic spline interpolant is unique.

5. (20%) (a) Write down the formulas corresponding to the following pseudocode and explain what does the pseudocode do? (b) Suppose `f(t,y)=yt(1-y)`. Count the floating point operations of the pseudocode in terms of n

```
input t0, y0, h, n
external f
for k=1 to n do
   t  = t0 + k*h
   t2 = t0 + 0.5*h
   v1 = f(t0, y0)
   v2 = f(t2, y0+0.5*h*v1)
   v3 = f(t2, y0+0.5*h*v2)
   v4 = f(t, y0+h*v3)
   y = (h/6)*(v1 + 2*(v2 + v3) + v4)
   y0 = y
   t0 = t
end for
```