數值分析(含程式設計)

1. (25 points) Suppose we want to solve a scalar nonlinear equation of the form $f(x) = 0$. Describe three different numerical methods to do that.

2. (25 points) Devise a numerical method to approximate the function $f(x) = 1/(1 + x^2)$ for $x \in [-5, 5]$ using, say, 10 nodal points. Give comments to the proposed method about its error and convergence behaviors.

3. (25 points) Devise two different ways to approximate the first derivative of a smooth function $f(x)$ at $x = x_0$ to second-order accurate. How would you do when f is only piecewise continuous and has discontinuities at a finite set of points ?

4. (25 points) Suppose we want to solve a linear system of equation $Ax = b$, where A is a square $n \times n$ matrix, b is a vector of n variables, and x is the unknown to be determined. Describe a numerical technique in details on how to go about that when n is not small.