臺灣大學數學系

九十二學年度碩士班甄試入學試題

數值分析(含程式設計)

Nov 29, 2002

1.

(45 points) Give a short definition of each of the terms:

1. [(a)] What is the fixed-point iteration for finding solution of $f(x) = 0$
2. [(b)] Condition number of a matrix A
3. [(c)] Singular value decomposition of a matrix A
4. [(d)] Newton interpolation formula
5. [(e)] Lagrange interpolation formula
6. [(f)] Cubic spline
7. [(g)] Jacobi method for solving large sparse linear system
8. [(h)] Gauss-Seidel method for solving large sparse linear system
9. [(i)] Zero stability of a linear multistep method for solving ordinary differential equation

2.

(25 points) Describe four different ways that may approximate a real-valued function $f(x) : \mathbb{R} \to \mathbb{R}$, to a desired accuracy. (In each case, you should state clearly the basic assumptions on f(x) so that the proposed method can be applied.)

3.

(30 points) The general (linear) least squares problem may be formulated as:

Given $A \in \mathbb{C}^{(m \times n)}$, $m \geq n$, $b \in \mathbb{C}^m$,

Find $x \in \mathbb{C}^m$ such that $\|b - Ax\|_2$ is minimized.

1. [(a)] (15 points) Show that the solution of this problem satisfies the equation

$$A^* A x = A^* b,$$

where A^* is the complex conjugate of A.

2. [(b)] (15 points) Describe two different algorithms that may be used to find the solution of the problem. Give comments to the possible advantage and disadvantage in using the algorithms you have proposed.