臺灣大學數學系
九十七學年度碩士班甄試試題
科目：線性代數　　2007.11.02

You should include in your answer every piece of computation and every piece of reasoning so that the corresponding partial credit could be gained.

(1) (30 points) Suppose that A is a 10×10 real matrix satisfying

$$A(A-1)(A-2)(A-3)(A-4)(A-5) = 0.$$

Does this imply that A is *diagonalizable*? Verify your answer.

(2) (25 points) Let A be an $n \times n$ *real symmetric* matrix. Show that there exists a *unique* real *symmetric* matrix B such that $B^9 = A$.

(3) (20 points) Let $u = (2,0,0,7,1,1,0,2)$, $v = (2,0,1,1,7,0,0,2)$ be two vectors in the Euclidean space $\mathbb{R}^8$ and let

$$P = \{\alpha u + \beta v \mid 0 \leq \alpha, \beta \leq 1\}$$

be the *parallelogram* spanned by u and v. Compute the area of P and verify your computation.

(4) (25 points) In $\mathbb{R}^3$ let C be a circle lying on a plane P which does not passing through the origin $(0,0,0)$. Let

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid (tx, ty, t) \in C \text{ for some } t \in \mathbb{R}\}.$$

Show that Γ is a *quadratic curve*. Namely, there is a polynomial $F(X,Y)$ of total degree 2 such that

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid F(x,y) = 0\}.$$

Is Γ an ellipse, a parabola or a hyperbola? Verify your answer.