臺灣大學數學系

八十八學年度碩士班甄試入學考試試題

線性代數

1. Find a 5 by 5 matrix A with entries in the field of real numbers such that the minimal polynomial satisfied by A is $X^5 + X^4 - 3X^2 + 1$.

2. Let $f : R^2 \to R^2$ be an isometry fixing the original point, that is, $|f(x) - f(y)| = |x - y|$ for all $x, y \in R^2$ and $f(0) = 0$. Then there exists an orthogonal matrix A (that is, $AA^t = I$, the identity matrix) such that $f(x) = Ax$ for all $x \in R^2$. Here, R denotesthe real numbers and the elements of R^2 are written as columns.

3. (乙組) Let

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix},$$

a 3 by 3 matrix with entries in the field of real numbers. Find an invertible matrix C such that

$$CAC^{-1} = \begin{pmatrix} 0 & a & b \\ c & 0 & d \\ e & f & 0 \end{pmatrix},$$

for some real numbers a, b, c, d, e, f.

4. (甲組) Let A be an n by n matrix with entries in the field of real numbers such that the trace of A is 0. Prove that there exists an invertible n by n matrix C such that $C^{-1}AC$ has only 0's on its main diagonal.