

臺灣大學數學系  
九十八學年度碩士班甄試試題  
科目：微積分與線性代數 2008.10.31

1. Let  $f$  be continuous on  $(a, b)$  and  $c \in (a, b)$ . Suppose that  $f$  is differentiable on  $(a, c) \cup (c, b)$ .
  - (i) Show that  $f'(c)$  exists if  $\lim_{x \rightarrow c} f'(x)$  exists.
  - (ii) Give an example to show that  $\lim_{x \rightarrow c} f'(x)$  need not exist even if  $f'(c)$  exists.
2. Let  $f(x) = \ln(1 + x + x^2)$  for  $x \in \mathbb{R}$ . Find its  $n$ -th derivative  $f^{(n)}(0)$  at  $x = 0$  for any natural number  $n$ .
3. Let  $D$  be the region bounded by the surface  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$  and the plane  $z = c$ , where  $a, b, c$  are positive numbers. Find the centroid of  $D$ .
4. Let  $F$  be a field and  $V$  an  $n$ -dimensional vector space over  $F$  where  $n > 1$ . Let  $v_1, \dots, v_n$  be a basis of  $V$  over  $F$  and  $T$  a linear transformation on  $V$  such that  $Tv_i = v_{n-i+1}$  for  $i = 1, \dots, n$ . Find the characteristic polynomial and the minimal polynomial of  $T$  and a basis of  $V$  over  $F$  consisting of eigenvectors of  $T$ .
5. (i) Show that  $4x^2 + 7y^2 + 7z^2 - 4xy - 4xz - 10yz \geq 0$  for all  $x, y, z \in \mathbb{R}$ .
  - (ii) Find a matrix  $A \in M_3(\mathbb{R})$  such that

$$A^2 = \begin{pmatrix} 4 & -2 & -2 \\ -2 & 7 & -5 \\ -2 & -5 & 7 \end{pmatrix}.$$