

臺灣大學數學系
九十七學年度碩士班甄試試題
科目：微積分與線性代數

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1. Let $f(x) = x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}$ for $x \in \mathbb{R}$. Determine the intervals of increase and decrease, the local extreme values of $f(x)$, the concavity, the inflection points and the asymptotes of the graph $f(x)$. Sketch the graph of $f(x)$.
2. Let $f(x) = \ln(x + (1+x^2)^{\frac{1}{2}})$ for $x \in \mathbb{R}$. Find its n -th derivative $f^{(n)}(0)$ at $x = 0$ for any natural number n .
3. Let $f(x, y) = |y - x^2|^{\frac{1}{2}}$ for $x, y \in \mathbb{R}$. Evaluate the double integral of $f(x, y)$ over $[-1, 1] \times [0, 2]$.
4. Let V be a finite dimensional vector space over an algebraically closed field F and S, T linear transformations on V . Show that ST and TS have the same characteristic values in F .
5. Let a_1, \dots, a_n be n elements in a field F and A the $n \times n$ -matrix with a_j^{i-1} as its (i, j) -entry, that is,

$$A = \begin{pmatrix} 1 & 1 & \dots & 1 \\ a_1 & a_2 & \dots & a_n \\ \vdots & \vdots & \ddots & \vdots \\ a_1^{n-1} & a_2^{n-1} & \dots & a_n^{n-1} \end{pmatrix}$$

Find the determinant of A .