臺灣大學數學系

九十五學年度碩士班甄試入學試題

微積分與線代

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1. $A = \begin{pmatrix} 12 & 3 & -7 \\ -18 & -5 & 10 \\ 17 & 4 & -10 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 & -3 \\ -10 & -3 & 6 \\ 5 & 1 & -4 \end{pmatrix}$. Are they similar to each other? If yes, $A = X^{-1}BX$, $X = \begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix}$. (20/100)

2. $y = ce^{-x}$ $(-\infty < c < \infty)$ is a family of curves. Find their orthogonal trajectories. (20/100)

3. T is a solid torus obtained by revolving a disk of radius $r = 1$ about the z-axis at a radius $R = 2$. Its moment of inertia I_z about the z-axis is $I_z = \int\int\int_T (x^2 + y^2)dxdydz = ?$

4. $\begin{cases} x &= t^2 - 2t \\ y &= t^3 + t^2 \\ z &= t - t^3 \end{cases}$ is a space curve. $t = 1$. $\vec{N}$=unit principal normal vector= $(?, ?, ?)$, $\vec{B}$=binormal vector= $(?, ?, ?)$. (20/100)

5. $\int_0^1 \int_0^{1-y} \frac{dxdy}{1-xy} = ?$ (20/100)