臺灣大學數學系

九十二學年度碩士班甄試入學試題

微積分與線性代數

Nov 29, 2002

1.

(20 points) Consider the following two-variable function

$$f(x,y)=\begin{cases} 2x^2y/(x^2+y^2) & \text{if} \quad (x,y)\neq(0,0) \\ 0 & \text{if} \quad (x,y)=(0,0). \end{cases}$$

(a)

(10 points) Is the function $f(x,y)$ continuous at $(x,y)=(0,0)$?

(b)

(10 points) Is the function $f(x,y)$ differentiable at $(x,y)=(0,0)$?

2.

(10 points) Find the shortest distance from the origin to the hyperbola $x^2+8xy+7y^2=225$.

3.

(10 points) Compute the surface integral

$$\int\int_{\mathcal{S}} \mathcal{F}\cdot\vec{N}\,d\sigma,$$

where $\mathcal{F}=\langle 2x,\ y,\ 2z\rangle$, $\mathcal{S}$ is the surface of the region bounded by the cylinder $x^2+y^2=16$, the xy plane, and the $z=2$ plane, and $\vec{N}$ is the outward unit normal vector on $\mathcal{S}$. 160π

4.

(10 points) Compute the line integral

$$\oint_{\mathcal{C}} \mathcal{F}\cdot\vec{T}\,ds,$$

where $\mathcal{F}=\langle z,\ 4x,\ 2z\rangle$, $\mathcal{C}$ is the bounding curve of the surface of the paraboloid

$z = 9 - (x^2 + y^2)$ above the xy plane, and $\vec{T}$ is the directed unit tangent vector along $\mathcal{C}$.

5.

(20 points)Given an $n \times n$ matrix A with real entries such that $A^2 = -I$. Prove the following statements about A.

(a)

(10 points) A is nonsingular.

(b)

(10 points) A has no real eigenvalues.

6.

(15 points)Find the Jordan canonical form of the following matrix

$$A = \begin{bmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & 1 \end{bmatrix}$$

7.

(15 points)Show that the transformation defined by T (x_1, x_2, x_3, x_4)= $(x_1 + x_2 - 3x_3 - x_4, 3x_1 - x_2 - 3x_3 + 4x_4, 0, 0)$ is a linear transformation on 4-dimensional linear space R^4,and find rank of T and kernel of T.