

臺灣大學數學系
九十七學年度碩士班甄試試題
科目：高等微積分

2007.11.02

1. (20 pts) Given $a, b, c > 0$. What is the volume enclosed by the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1?$$

2. (20 pts) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable. Suppose that $f(0) = 0$ and $f'(x) > f(x)$ for $x \in \mathbb{R}$. Can $f(c) \leq 0$ for some $c > 0$? Prove or disprove your answer.

3. (30 pts) Consider the system of equations

$$\begin{aligned} 3x + y - z + u^4 &= 0 \\ x - y + 2z + u &= 0 \\ 2x + 2y - 3z + 2u &= 0. \end{aligned}$$

- (i) Prove that for some $\epsilon > 0$, the system can be solved for (x, y, u) as a function of $z \in [-\epsilon, \epsilon]$, with $x(0) = y(0) = u(0) = 0$. Are such functions $x(z), y(z), u(z)$ continuous? differentiable? unique? (15 pts)
- (ii) Can the system be solved for (x, y, z) as a function of $u \in [-\delta, \delta]$ for all $\delta > 0$? (15 pts)

Prove or disprove all your answers.

4. (15 pts) Suppose that $f : [0, 1] \rightarrow \mathbb{R}$ satisfies

$$\lim_{x \rightarrow c} f(x) \in \mathbb{R}, \quad \forall c \in (0, 1),$$

and $f(0+) = \lim_{x \rightarrow 0+} f(x), f(1-) = \lim_{x \rightarrow 1-} f(x) \in \mathbb{R}$. Can f be Riemann integrable on $[0, 1]$? Prove or disprove your answer.

5. (15 pts) Let $u : \mathbb{R}^2 \rightarrow \mathbb{R}$ be analytic and nonconstant. Suppose

$$u_y = u_{xx} \quad \text{for } x, y \in \mathbb{R}.$$

Can the function u have a local maximum point at $(x_0, y_0) \in \mathbb{R}^2$? Prove or disprove your answer.