臺灣大學數學系

九十二學年度碩士班甄試入學試題

高等微積分

Nov 29, 2002

1.

$\int\int_{R^2} \frac{1}{[2+(3x-y+1)^2+(x+y-1)^2]^2}$ dxdy=?(25/100)

2.

Let $f_k(x) = a_{k,0} + a_{k,1}x + a_{k,2}x^2 + ... + a_{k,k}x^k$

$a_{k,l} \geq 0$, $\sum_{l=0}^{k} a_{k.l} = 2$, Show that there is a subsequence

$\{f_{k_j}(x)\}$of $\{f_k(x)\}$ which converges uniformly on $0 \leq$x$\leq 1/2$. (25/100)

3.

Let f(x) be a continuous function on $[0, \infty)$ and there exist constants $M > 0$, $P > 0$

such that |f(x)| $\leq MX^p$ for all $X > 1$. Compute

$$\lim_{n\to\infty} n \int_0^\infty f(x)e^{-nx}dx$$

(justify your answer) (25/100)

4.

Suppose $A(x) = \begin{pmatrix} a_{11}(x) & a_{12}(x) \\ a_{21}(x) & a_{22}(x) \end{pmatrix}$ is a 2×2 matvix of complex-valued functions,

X $\in$R. $a_{ij}(x)$ are C^1 in a nerghborhood of x_0 $\in$R. Assume that $\lambda_1(x_0)$ and $\lambda_2(x_0)$

are eigenvalues of $A(x_0)$, $\lambda 1(x_0) \neq \lambda_2(x_0)$ Show that near x_0 there exists a matrix

function P(X) with C' elements and two scalar C' functions $\lambda_1(x)$, $\lambda_2(x)$ such that

$P^{-1}(x)A(x)P(x) = \begin{pmatrix} \lambda_1(x) & 0 \\ 0 & \lambda_2(x) \end{pmatrix}$ Give an example to show that this is not true

of $\lambda_1(x_0) = \lambda_2(x_0)$ (25/100)