

# 台灣大學數學系

## 九十二學年度第二學期博士班資格考試題

### 幾何與拓樸

May 8, 2004

[\[回上頁\]](#)

**25 points for each problem.**

1. Compute Homology groups  $H_*(S^n, \mathbb{Z})$ , where  $S^n$  is the  $n$ -dimensional sphere.
2. Show that tangent bundles of Lie groups are trivial.
3. Show that Riemannian manifolds with non-positive sectional curvature have no conjugate points, by carrying out the proof of the related comparison theorem of Sturm type.
4. For the disk  $M = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$  with the metric  $ds^2 = \frac{4}{(1-x^2-y^2)^2}(dx^2 + dy^2)$ , show that  $M$  is a complete Riemannian manifold with sectional curvature being  $-1$  everywhere.

[\[回上頁\]](#)