臺灣大學數學系

九十一學年度第一學期碩博士班資格考試題

幾何（Geometry ）

Sept 14, 2002

1.

X= $\{(cos\theta, sin\theta, z)|0 \leq \theta \leq 2\pi, -1 < z < 1\}$ is a cylinder and is an orientable surface $\pi : X \to Y$ is a covering map . Can you prove that Y is an orientable surface too ? (25/100)

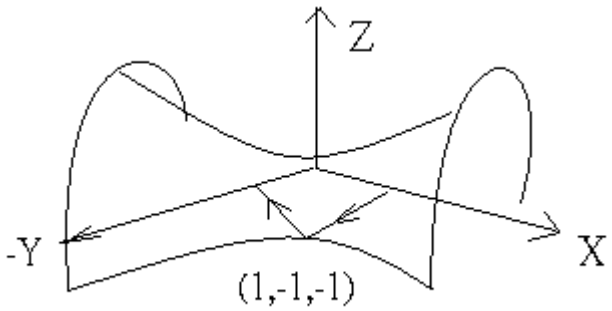
2.

$w = xdy + ydz + zdw + wdx$ is a differential in R^4. $\Omega = dw$ is a differential 2-form ? Is Ω a closed 2-form ? Is Ω a symplectic 2-form ?(25/100)

3.

$z = xy$ is a hyperbolic paraboloid. $v = (1, 0, 0)$ is a tangent vector at $(x, y, z) = (0, 0, 0)$ Parallel translate v around a loop $(0, 0, 0) \to (1, 0, 0) \to (1, -1, -1) \to (0, -1, 0) \to (0, 0, 0)$ consisting of 4 segments .

Find the ending vector $v = (?, ?, 0)$ (25/100)



4.

$z = xy, x^2 + y^2 = 1$ is a curve in R^3. At the point $(x, y, z) = (1, 0, 0)$ curvature $k = ?$, torsion $\tau = ?$ (25/100)