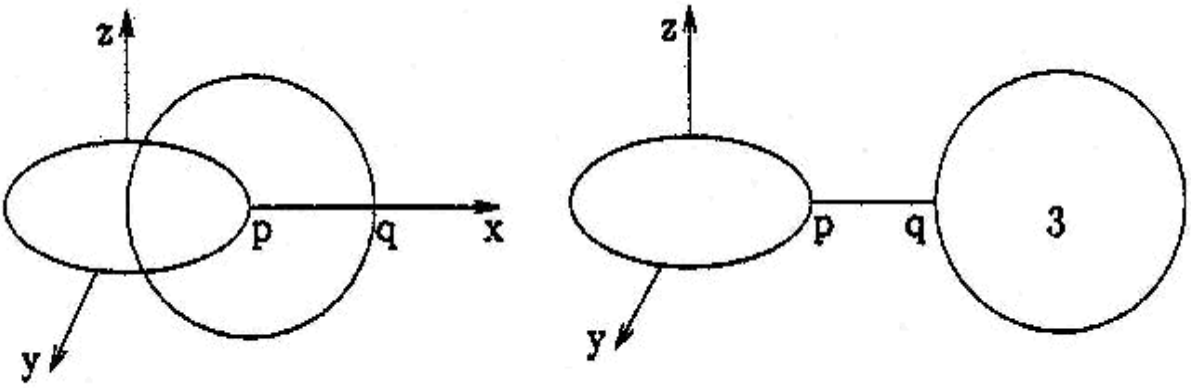
臺灣大學數學系

九十學年度第一學期碩博士班資格考試試題

幾何

1. $\mathbb{R}^4 = \{(x,y,z,w) |\ x,y,z,w \in \mathbb{R}\}$. Is $\mathbb{R}^4$ a simply connected manifold? Can you find a differential 2-form w on $\mathbb{R}^4$ so that $w \wedge w$ is a differential 4-form but $w \wedge w \neq 0$? (25/100)

2. 

$\mathbb{R}^3$ ∋ $p = (1,0,0)$, $q = (2,0,0)$, $\overline{pq}$ =line interval.

$C_1 = \{x^2 + y^2 = 1,\ z = 0\}$,

$C_2 = \{(x-1)^2 + z^2 = 1,\ y = 0\}$,

$C^3 = \{(x-3)^2 + z^2 = 1,\ y = 0\}$,

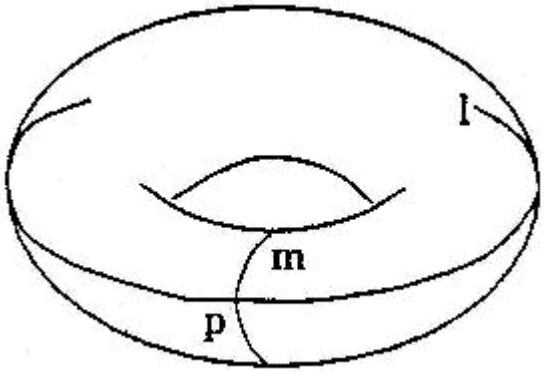
$X = \overline{pq} \cup C_1 \cup C_2$,

$Y = \overline{pq} \cup C_1 \cup C_3$.

Do X and Y have the same fundamental group?(25/100)

3. $x^2 + y^2 = z^2$ is a cone, $x + y + x = 12$ is a plane. Their intersection is a conic section C. Is C an ellipse or hyperbola? $p = (3,4,5)$ is a point on C, at this point, C has curvature K =? torsion τ =? (25/100)

4.



$T = \{(x - \frac{x}{\sqrt{x^2+y^2}})^2 + (y - \frac{y}{\sqrt{x^2+y^2}})^+ z^2 = \frac{1}{4}\}$ is a torus, $p = (\frac{3}{2},\ 0,\ 0)$ is a point on T. l =lattitude circle $= \{x^2 + y^2 = \frac{9}{4},\ z = 0\}$ m =meridian circle $= \{(x-1)^2 + z^2 = \frac{1}{4},\ y = 0\}$. Can you find a tangent with $\vec{V}$ at p so that its parallel translation along l lack to p is different from its parallel translation along m lack to p ? $\vec{V} = (?,\ ?,\ ?)$ (25/100)