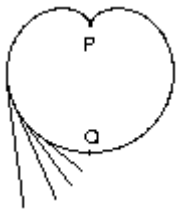
臺灣大學數學系

八十九學年度第二學期碩博士班資格考試試題

幾何

1.



Let P be the pole of polar coordinate. $r = 1 - \sin\theta$ is a cardioid.

$Q = (x, y) = (0, -2)$. Arclength $\overset{\frown}{PQ} = L =$? A string of length L has its one end fixed at P and winds around the cardioid so that its other end generates the involute of the cardioid. Is this involute a cardioid, too? (25/100)

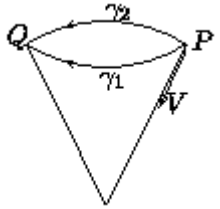
2.

Can you find a surface in $\mathbb{R}^3$ passing through the origin $(x, y, z) = (0, 0, 0)$ so that both its mean curvature H and Gauss curvature K vanish at the origin yet the surface is not a plane? If not, explain why not. (25/100)

3.

Can you find a closed differential 2-form ω in $\mathbb{R}^3 - (0, 0, 0)$ which is not exact? If yes, $\omega = ?dx \wedge dy + ?dy \wedge dz + ?dz \wedge dx$ (25/100)

4.



$\text{Cone} = \{x^2 + y^2 = z^2\},\ P = (1, 0, 1), Q = (-1, 0, 1)$

$\gamma_1 = \text{cone} \cap \{z = 1\} \cap \{y \geq 0\}$

$\gamma_2 = \text{cone} \cap \{z = 1\} \cap \{y \leq 0\}$

$\vec{V} = (-1, 0, -1)$ is a tangent vector to the cone at P.

Parallel translate $\vec{V}$ from P to Q along $\gamma_1 = (?, ?, ?)$. If we translate along γ_2 instead of γ_1, do we get the same vector? (25/100)