臺灣大學數學系

八十八學年度第二學期碩博士班資格考試試題

幾何

1. Let X, Y be two topological spaces, and f_0, f_1 are two continuous maps from X to Y
.
    1. (5pts) What does that f_0 is homotopic to f_1 mean ?
    2. (5pts) How to define the first fundamental group of X, $\Pi_1(X, x_0), x_0 \in X$?
    3. (5pts) When will we call that X, Y are of the same homotopy type?
    4. (5pts) If X and Y are both arcwise connected, and of the same homotopy type, what is the relation between $\Pi_1(X)$ and $\Pi_1(Y)$? Give the reason briefly.
    5. (10pts) $\Pi_1(\mathbb{R}^2 - \{P_1, P_2\}) =?$
2. $X(u,v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, v^2 - v^2)$, $(u,v) \in \mathbb{R}$. Denote the image of $X(u,v)$ by S.
    1. (10pts) Find the first fundamental form of S.
    2. (10pts) Find the second fundamental form of S.
    3. (10pts) Compute the Gaussian curvature and mean curvature of S.
    4. (10pts) Compute the Christoffel symbols for the first fundamental form.
    5. (5pts) Write down the geodesic equations on S with explicit coefficients.
    6. (5pts) Let $p \in S$, T_p the tangent plane of S at p, T_p^ε a plane parallel to T_p with distance ε. Sketch roughly the picture of $S \cap T_p^\varepsilon$ near p up to the first order.
    7. (10pts) Let D be the unit Disk on $\mathbb{R}^2$. Use Gauss-Bonnet Theorem to find the total geodesic curvature $\int_\gamma k_g ds$ on S, where $\gamma(\theta) = X(\cos\theta, \sin\theta)$ considered as the boundary of $X(D)$.
3. (10pts) State the Gauss-Bonnet Theorem (both local and global) with a clear definition for all the symbols in the formula.