臺灣大學數學系

八十八學年度第一學期碩博士班資格考試試題

幾何

A.

What is the fundamental group of the following space? (*Fig.1*) Describe the universal covering space of it.

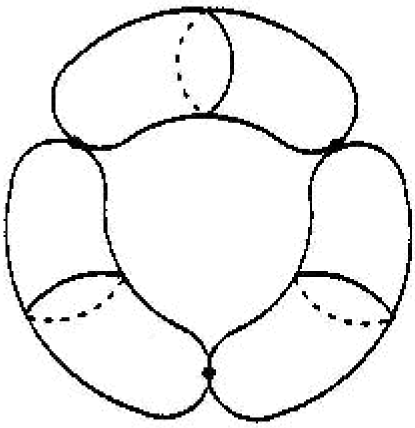

Fig. 1

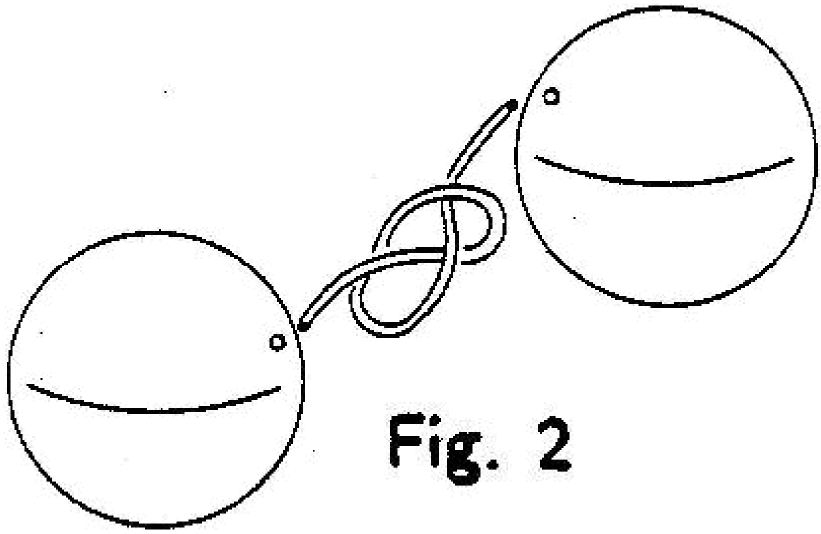

Fig. 2

B.

As *Fig.2*, there are two round spheres, and on each of them we remove a relatively small disk. We smoothly connect the two holes by a tube (i.e. diffeomorphic to a cylinder). Prove that no matter what the tube looks like, there is at least one point on it whose Gaussian curvature is negative. (*Hint.* Gauss-Bonnet theorem)

C.

Given a surface Σ with two metrics dg and dh. Suppose there is a function f such that

$$dg = e^f \cdot dh, \qquad i.e. \quad \underline{conformal} \quad to \quad each \quad other$$

What is the relation between the two Gaussian curvatures of a point with respect to the two metrics.

Remark. In tensor notation, the relation is $g_{ij} = e^f \cdot h_{ij}$, $i, j = 1, 2$.

D.

Let Σ be a surface in $\mathbb{R}^3$ and $\nu : \Sigma \to \mathbf{S}^2$ be the Gauss map, where $\mathbf{S}^2$ is the unit sphere. Suppose γ is a geodesic on Σ, is $\nu(\gamma)$ a portion of a great circle of $\mathbf{S}^2$? Make a judgement and state your reasoning.

E.

Which geometric notion of the following list is intrinsic, i.e. determined by the metric only.

(1) Gaussian curvature of a surface.
(2) mean curvature of a surface.
(3) area of a region on a surface.
(4) angle between two intersected curves.
(5) length of a curve.
(6) geodesic curvature of a curve on a surface.
(7) curvature κ of a space curve.
(8) torsion τ of a space curve.

Answer the question with a short explanation.