

# 臺灣大學數學系

## 八十七學年度第二學期碩博士班資格考試試題

### 幾何

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- (25%)  $K = \text{klein bottle} = \{(e^{i\theta}, \varphi) \in \mathbb{C} \times \mathbb{R}\} / \sim$ , where the equivalence relation  $\sim$  is generated by  $(e^{i\theta}, \varphi) \sim (e^{i(\theta+\pi)}, \varphi + 1)$ . Can you find an immersion of  $K$  into  $\mathbb{R}^3$ ? If yes,  $x = x(\theta, \varphi) = ?$ ,  $y = y(\theta, \varphi) = ?$ ,  $z = z(\theta, \varphi) = ?$
- (25%)  $z = x^2 - y^2$  is a hyperbolic paraboloid. Is the curve  $y = 0$  a geodesic? If yes, parallel translate the vector  $\vec{v} = (1, 1, 0)$  at the origin  $(x, y, z) = (0, 0, 0)$  along  $y = 0$  to the point  $(x, y, z) = (1, 0, 1)$ .
- (25%)  $N = \mathbb{R}^3 - (x\text{-axis}) - (y\text{-axis}) - (z\text{-axis})$ . Is  $N$  a connected space? If yes, is its fundamental group  $\pi_1(N)$  an abelian group?
- (25%) Can you find a compact surface  $M$  in  $\mathbb{R}^3$  so that its mean curvature  $H(M) \equiv \text{constant} = 0$ ? If not, can you find a compact surface  $N$  in  $\mathbb{R}^3$  so that its gauss curvature  $K(M) \equiv \text{constant} = 0$ ?

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