

臺灣大學數學系

八十九學年度第二學期碩博士班資格考試試題

分析

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一、

(a)

Give an example which shows that the image of a measurable set under a continuous transformation may not be measurable.

(b)

If f is absolutely continuous on $[a, b]$, show that the image under f of any measurable subset of $[a, b]$ is measurable.

二、

Let f be a measurable real-valued function on $[a, b]$. Show that given $\delta > 0$, there is a continuous function g on $[a, b]$ such that the Lebesgue measure

$|\{x : g(x) \neq f(x)\}| < \delta$. Can you do the same on the interval $(-\infty, \infty)$? (Lusin's Theorem)

三、

(a)

Let f be a nonnegative function which is integrable over a set $E \subset \mathbb{R}^1$. Show that given $\varepsilon > 0$, there is a $\delta > 0$, such that for every set $A \subset E$ with Lebesgue

measure $|A| < \delta$ we have $\int_A f(x) dx < \varepsilon$.

(b)

Let $\{f_n\}$ be a sequence of measurable functions on a set E . If $f_n \rightarrow f$ in measure and there is an integrable function g such that $|f_n| \leq g \forall n$, show that

$\int_E |f_n - f| \rightarrow 0$. (Use (a))

四、

(a)

Let f be a continuous function on $[a, b]$, and

$$\int_a^b x^n f(x) dx = 0 \quad \forall n = 1, 2, 3, \dots. \text{ Show that } f \equiv 0.$$

(b)

If f is a Lebesgue integrable function on $[a, b]$ and $\int_a^b f(x)\varphi(x) dx = 0$ for all continuous function φ on $[a, b]$, what can be said about f , give the reasons.

五、

$$\text{Let } f(x) = \begin{cases} 0 & \text{for } x = 0 \\ x^\alpha \sin(x^{-\beta}) & 0 < x \leq 1 \text{ and } \alpha > 0, \beta > 0 \end{cases}$$

For what values of α, β is the function $f(x)$

- (a) continuous?
- (b) of bounded variation?
- (c) absolutely continuous?

(以上五題選做四題)

六、

(a)

Evaluate the integral $\int_{-\infty}^{\infty} \frac{\cos x}{a^2 + x^2} dx$, a is a positive constant.

(b)

Let f be a continuous function on $[a, b]$, show that $g(z) = \int_a^b e^{-zt} f(t) dt$ is an entire function of z (analytic in the entire complex plane).

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