

臺灣大學數學系

八十八學年度第二學期碩博士班資格考試試題

分析

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Part A: There are 7 problems in this section, do any 4 problems from them.

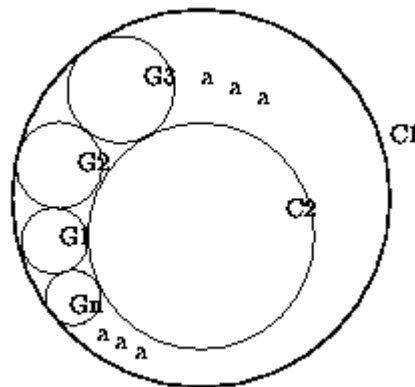
- $x \in (0, 1)$, Call x a lucky number if there are infinite many 8's in the decimal expansion of x . Call x a really lucky number if there are infinitely many 8's, infinitely many 88's, infinitely many 888's, infinitely many 8888's ... in the decimal expansion. Show that the set of really lucky number is measurable and find its measure.
- $f \in L^1(a, b)$, given $\varepsilon > 0$, there exists $\delta > 0$ such that $\int_E |f(x)| dx < \varepsilon$ whenever $m(E) < \delta$ for all measurable subset $E \subset (a, b)$. Where $m(E)$ is the Lebesgue measure of E .
True or false? Justify your answer.
- Let $f(x) = \sum_{n=0}^{\infty} \frac{\cos nx}{2^n}$
 - Does $f \in C^0(\mathbb{R})$?
 - Does $f \in C^1(\mathbb{R})$?
 - Does $f \in C^\infty(\mathbb{R})$?
 - Is f analytic on \mathbb{R} ?
- Let $f \in L^1(0, \pi)$, find the following limits:
 - $\lim_{n \rightarrow \infty} \int_0^\pi f(x) \sin nx dx$
 - $\lim_{n \rightarrow \infty} \int_0^\pi f(x) |\sin nx| dx$
- Is the closed unit ball in l^2 compact ?
 - Let $E = \{(x_1, x_2, x_3, \dots) \mid |x_i| \leq \frac{1}{n}\}$. Is E compact in l^2 ?
- Does there exist a bounded linear operator $T : L^2(0, 1) \rightarrow L^2(0, 1)$. Whose range is the set of all polynomials ?
- Show that $L^p(0, 1)$ is complete for $1 \leq p < \infty$.

Part B: There are 4 problems in this section, do any 2 problems from them.

- Evaluate the following integrals:

(a) $\int_{|z|=r} \frac{|dz|}{|z-a|^2}$ (b) $\int_{|z|=r} \frac{d\theta}{z-\alpha}$, $|\alpha| \neq r$ (c) $\int_{|z|\leq 1} \frac{dx dy}{z-\alpha}$

2. If $f(z)$ is analytic in $|z| \leq 1$ and satisfies $|f(z)| = 1$ on $|z| = 1$. Show that $f(z)$ is a rational function.
3. Let Ω be the unit disk minus the real interval $(0, 1)$. Find a harmonic function u in Ω so that $u = 1$ on $|z| = 1$ and $u = 0$ on $(0, 1)$.
4. Let C_1, C_2 , be two circles in the plane and suppose that C_1 lies inside C_2 . Let Γ_1 be a circle in the annular region which is tangent to C_1 and C_2 somewhere and let Γ_2 be the circle tangent to C_1, C_2 and Γ_1 ; Γ_3 be circle tangent to C_1, C_2 and Γ_2 ; \dots . Suppose that the n th circle Γ_n tangent to Γ_1 (as shown in the figure). Show that no matter how the first circle Γ_1 take place in the annular region, the last circle Γ_n is always tangent to Γ_1 and n is independent of Γ_1 .



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