Course Syllabuses*

Department of Mathematics

This document contains the descriptions of the (mainly undergraduate) courses which are required in the department. It explains what are the goals and what will be covered in each course. In principle, the materials in the description of each (one-semester) course are supposed to occupy only 12∼13 weeks out of 17 weeks and, in this case, some selected topics are also provided.

*May 7, 2015

Contents

Calculus (I, II)

Differentiation studies the rate of change; while integration reveals the effect of accumulation. These two ingredients (rate of change and effect of accumulation) are used to characterize tons of principles, phenomena and criterions surrounding the daily life of everybody. The history of development of calculus somehow reflects part of the most exciting experience that all human beings who had ever lived on earth shared. The mission of calculus on campus has at least the following concerns. (1) The philosophy of the theory of calculus is logically rigorous. The first benefits that a student who receives this training is to cultivate his/her solid logic thinking. In other words, he/she would have a better idea to make logically reasonable judgement and avoid to make funny mistakes. (2) The history of calculus is both heuristic and creative. With this cultivation, students are able to broaden their insights and get inspirations. This might give them motivations to devote themselves to the scientific research. (3) The philosophy and the skills that students learn from this course give direct applications to a lot of scientific work (including natural science and social science).

Nowadays, the use of network and scientific computing is getting more and more important. If time permits, we may integrate a few numerical computation programming sessions in this class to train students the basic/important ingredients of calculus including (a) input/output (b) do loop (c) if selection (d) graphics and (e) error control.

The concrete contents of this course are listed as below.

Contents

1. Basic logic and set theory: concept, notation and examples
2. The ways to do rigorous mathematical proofs
3. Review of basic algebraic formulae, inequalities and elementary functions
4. Limits and continuity
5. Differentiation: Fermat's theorem, Rolle's theorem, mean value theorem, Cauchy mean value theorem, extreme values, product rule, chain rule.
6. Series and integration: Riemann sum, fundamental theorem of calculus
7. Construction of transcendental functions and their differentiations: log function, exp function.
8. Techniques of integration
9. Convergence of sequence: ratio test, root test, Abel test, Dirichlet test
10. Taylor expansion theorem and its applications: convergence of sequence of functions, radius of convergence
11. Applications: logistic model, Newton's heat conduction law, Kepler's laws of planetary motion, Newton's method for roots solving.

12. Fourier series and its applications
13. Parametric curves and polar curves.
14. Functions of several variables and their derivatives
15. Developments and applications of the differential calculus
16. Multiple integrals
17. Implicit functions
18. Technique of integration from the point of view of multivariable calculus
19. Relations between surfaces and volume; Green's and Stokes' theorems
20. Selected topics: Fourier transform; Differential equations

References

- Richard Courant and Fritz John, *Introduction to Calculus and Analysis (I) (II).*
- James Stewart, *Calculus : Early Transcendentals.*

Introduction to Analysis (I, II)

The aim of this course is to introduce the solid foundation of analysis. On the one hand, we introduce the most important concepts, theories, machinery and examples to train students who want to devote themselves in mathematical research. On the other hand, this course is also prepared for students to employ mathematical analysis to other disciplines. To achieve the above two goals, we shall present abstract theorems in reasonable concrete forms (we shall avoid to generalize theorems to too abstract forms in our course). On the other hand, we encourage students to work with concrete (important) examples. We expect that the students can both cultivate themselves with the understanding of theorems and the capability of calculation.

Contents

1. The topology of Euclidean space
2. Continuous mappings
3. Uniform convergence
4. Differentiable mappings
5. The inverse and implicit function theorems
6. Integration
7. Fubini's theorem and change of variable formula
8. Fourier series

References

- Jerrold E. Marsden and Michael J. Hoffman, *Elementary classical analysis.*
- Walter Rudin, *Principle of mathematical analysis.*
- Richard Courant and Fritz John, *Introduction to Calculus and Analysis.*

Introduction to Ordinary Differential Equations

The laws of nature are expressed as differential equations. Scientists and engineers must know how to model the world in terms of differential equations, and how to solve them and interpret the solutions. This course focuses on those basic equations, including linear differential equations and lower-order nonlinear equations. The models include population dynamics from ecology, oscillations from mechanics, periodic solutions from planet-star systems and biological systems, and chemical reaction models from chemistry.

Contents

1. Modeling population dynamics with first-order equations
2. Second-order linear equations, the spring-mass systems, the circuit systems, damping, forcing and resonance, stability
3. Linear systems with constant coefficients, method of Laplace transform
4. Nonlinear systems on the plane: pendulum, planet-star system, Lotka-Volterra system, van der Pole oscillators, and selective topics on phase plane analysis, Lyapunov function, stability analysis, Poincaré map
5. Selective topics about existence, uniqueness theory and error analysis for simple numerical schemes for ordinary differential equations.

References

- James R. Brannan and William E. Boyce, Differential Equations, An Introduction to Modern Methods and Applications
- William E. Boyce and Richard C. DiPrima, Elementary Differential Equations and Boundary Value Problems

Introduction to Partial Differential Equations

Partial differential equations (PDEs) are widely used by scientists to investigate the fundamental processes of the natural world. In this course, we introduce both the mathematical theories of PDEs and the scientific PDE models. We start with first order equations and spend most time in the second order equations. We shall give a concise introduction to the most important three types of PDEs: Elliptic equations, parabolic equations and hyperbolic equations.

Contents

1. First order linear equations
2. Flows, vibrations and diffusions
3. Initial and boundary conditions
4. Waves and diffusions
5. Reflections and sources
6. Dirichlet boundary condition, Neumann boundary condition and Robin boundary condition
7. Laplace equation
8. Maximum principle
9. Green's functions
10. Fourier series, heat equations and wave equations

References

- Fritz John, *Partial differential equations.*
- Walter A. Strauss, *Partial differential equations.*
- Lawrence C. Evans, *Partial differential equations.*

Linear Algebra (I, II)

Linear Algebra (I)

The concepts of linear algebra appear as a fundamental language in a large part of social and natural sciences in an essential way. Linear algebra also provides the first step toward understanding and manipulating abstract algebraic systems. The two-semester course covers basic and standard concepts of linear algebra needed for students in mathematics department. Explicit goals in the first semester include familiarize with the main objects – linear spaces (possibly equipped with additional structures), and the relations between them or upon themselves – linear transformations and their matrix representatives, kernels, quotients, dual spaces, eigenvalues, etc.

Contents

1. Vector spaces, basis, dimension
2. Linear transformations
3. Kernel, quotient, dual
4. Determinant
5. Characteristic polynomial, eigenvalues, eigenvectors, diagonalization

References

- S. Friedberg, A. Insel and L. Spence, *Linear Algebra*, 4th Edition. 2003.
- K. Hoffman and R. Kunze, *Linear Algebra*, 2nd Edition. 1971.

Linear Algebra (II)

This course covers basic and standard concepts of linear algebra needed for students in mathematics department. The basic materials in the second semester include the structure theorem of linear endomorphisms (the Jordan and rational canonical forms) and the study of spaces with product structure and their applications. Linear transformations provide first approximations to general functions, and hence the canonical forms of them provide the very first understanding of general phenomena, e.g. in the theory of differential systems. On the other hand, linear spaces with a product structure appears naturally in geometry or quantum mechanics.

Additional topics may include an introduction to multilinear algebra or to linear groups.

Contents

1. Jordan and rational canonical forms; minimal polynomial
2. Spaces with (inner or Hermitian) product structure
3. Natural operators (normal, self-adjoint, unitary, orthogonal operators) and their structures
4. Bilinear and quadratic forms; their classifications over the field of real numbers

References

- S. Friedberg, A. Insel and L. Spence, *Linear Algebra*, 4th Edition. 2003.
- K. Hoffman and R. Kunze, *Linear Algebra*, 2nd Edition. 1971.

Introduction to Algebra (I, II)

Algebra structures appear as common abstract structures of underlying symmetries in various areas of mathematics, and conversely the language and concepts of algebra have integrated naturally into all modern mathematics, both pure and applied, as useful tools to describe various phenomena in a systematical way. The goal of the two-semester course is to equip students the knowledges of the most basic objects (groups, rings, and fields) in abstract algebra, their fundamental classification theorems (e.g. Sylow theorems) and their interrelations (e.g. Galois theory). After the course, students shall feel comfortable in using language from algebra in all areas of sciences. More precisely, we shall cover

1. Group: Sylow theorems, group presentation, composition series
2. Ring: Chinese remainder theorem, Eisenstein criterion, division algorithm, PID, UFD, integral domain, field of fraction
3. Fields: Geometric construction/ruler-compass (including regular n-gon), insolvability of quintic, Galois theory
4. At least one of: Finitely generated abelian groups; modules over a PID

and students shall be able to

1. Classify groups of lower orders
2. Use Chinese remainder; manipulate polynomial rings
3. Understand the insolvability of a general quintic equation over the rationals
4. Understand the structure of finitely generated abelian groups

References

- M. Artin, *Algebra*, 2nd Edition. 2010.
- D.S. Dummit and R.M. Foote, *Abstract Algebra*, 3rd Edition. 2003.

Introduction to Probability Theory

Probability theory, originated in the consideration of games of chance, is the language to study commonly observed random phenomena. It has become a fundamental tool used by nearly all scientists, including engineers, econometricians, industrialists, jurists, medical practitioners, physicists, statisticians, etc. The main objective of this course is to provide students, who possess the prerequisite calculus background, with a solid mathematical treatment of the fundamental concepts and techniques of probability theory. Another goal is to demonstrate the many diverse possible applications of the subject through many examples.

Contents Axioms of probability, conditional probability, independence, random variables, jointly distributed random variables, expectation, moment generating functions, limit theorems, Poisson processes, Markov chains.

References

- Ross, S. (2013). *A First Course in Probability*, 9th edition, Pearson Education Inc.
- Durrett, R. (2009). *Elementary Probability for Applications*. Cambridge University Press.
- Durrett, R. (1994). *The Essentials of Probability*, Duxbury Press.

Introduction to Geometry

Surfaces in the three-space form the basis of modern differential geometry. They are also the most important examples in modern differential geometry. The main goal of this course is to familiarize with the concepts of distance and shortest path on a surface, how to describe the non-flatness of a surface precisely, and how the non-flatness of a surface is related to its global shape. The main tools of this course are linear algebra and multi-variable calculus, especially in two dimensions/variables.

Contents

1. Curvature and torsion of curves.
2. Regular surface and its tangent plane.
3. First fundamental form and the Gauss map.
4. Geodesic, covariant derivative and parallel transport.
5. Curvature of surfaces: intrinsic and extrinsic meanings.
6. Global property of curvature: the Gauss-Bonnet theorem.

The above materials are supposed to occupy only 12∼13 weeks out of 17 weeks.

References

1. M.P. do Carmo, *Differential geometry of curves and surfaces*, 1976.
2. B. O'Neill, *Elementary differential geometry*, 2nd edition, 2006.
3. J. Oprea, *Differential Geometry and its Applications*, 2nd edition, 2007.

Introduction to Complex Analysis

Complex function theory is a valuable tool used in many branches of pure, applied mathematics and natural sciences, including geometry, number theory, partial differential equations and various topics in physics and engineering. A basic course shall enable students to understand the concept of complex analyticity, to use residue calculus for evaluation of integrals and to learn some additional topics (depending on available time) selected from Riemann mapping theorem, special functions, prime number theorem, complex dynamical systems, etc.

Contents

- Analytic functions of a complex variable and power series,
- Cauchy's integral theorem,
- maximum modulus principle and open mapping theorem,
- singularities of analytic functions and Laurent series,
- residue theorem and its applications: argument principle, Rouche's theorem and the evaluation of integrals,
- analytic continuation,
- conformal mapping (on basic domains) and Schwarz lemma,
- Weierstrass infinite products,
- harmonic functions and the Dirichlet problem.
- Selected topics: Riemann mapping theorem, Schwarz-Christoffel integral, complex dynamical systems, prime number theorem, elliptic functions, etc.

Selected textbooks

1. Stein, E.M., Shakarchi, R., "Complex analysis"
2. Lang, S., "Complex analysis", GTM 103
3. Ahlfors, L., "Complex analysis"

Introduction to Computational Mathematics

This is an introductory course to scientific computing. We will discuss mathematical principles, numerical algorithms, and software development for solving realistic scientific and engineering problems. One distinguishing feature of the course is "learning-by-doing". In addition to deductive learning (give a formula and then follow the instructions), we also emphasize inductive learning by asking why things are (or are not) running in a certain way and asking students to invent solutions. Students are expected

- to obtain an intuitive and working understanding of basic numerical methods,
- to gain appreciation of the concept of computational error and of the need to analyze and predict it,
- to implement numerical methods by using a computer language (mainly MATLAB), and
- to develop teamwork experience plus oral communication and writing skills.

Contents This introduction level course covers basic scientific computing topics, including

- Nonlinear equations,
- Approximation,
- Numerical differentiation and integration,
- Numerical linear algebra (linear systems and eigenvalues problems),
- Numerical optimization,
- Numerical differential equations, and
- Others.

References

1. "Numerical Analysis" by Richard L. Burden and J. Douglas Faires (9th Edition)
2. "Numerical Computing with MATLAB" by Cleve Moler
3. "Scientific Computing with MATLAB and Octave" by Alfio Quarteroni, Fausto Saleri and Paola Gervasio (4th Edition)
4. "Introduction to Scientific Computing" by Charles F. van Loan (2nd Edition)
5. "Scientific Computing: An Introductory Survey" by Michael T. Heath (2nd Edition)
6. "Numerical Analysis: Mathematics of Scientific Computing" by David Kincaid and Ward Cheney (3rd Edition)
7. "Numerical Mathematics" by Alfio Quarteroni, Riccardo Sacco and Fausto Saleri.
8. "MATLAB Guide" by Desmond J. Higham and Nicholas J. Higham (2nd Edition)

Modern Algebra (I, II)

Algebra provides common and powerful language in describing symmetry in all areas of science. This course provides an opportunity to learn the basics of algebra, both commutative and non-commutative, especially to those who did not have them in their undergraduate study and would like to know them as tools or as research topics toward their graduate study.

Contents

- Modules over PIDs, tensor product, localization, primary decomposition, chain conditions
- Algebraic extensions of fields, algebraic closure, transcendental extensions
- Selected: Representation of finite groups, semisimple algebras

Goals

1. Basics and structures of groups, various rings and fields
2. Galois theory, manipulating with finite fields
3. Comfortable with modules, their tensor products, localizations, structure theorem of finitely generated modules over PIDs.
4. Structure theorems of simple algebras
5. Basics of representation theory of finite groups, character tables

Differential Geometry (I, II)

Modern differential geometry is a discipline, which uses (multi-)linear algebra and differential calculus to study problems in geometry.

Roughly speaking, the following concepts and their theories are the main theme of this course.

- A *Manifold* is the higher dimensional generalization of a surface, over which we can do differential calculus.
- A *Metric* gives the notion of distance, angle, volume, etc. on a manifold.
- Over a *Vector Bundle* we can study "twisted" functions on a manifold, and obtain more fruitful stories for a manifold. These twisted functions are sort of functions with pole/infinity.
- *Topology* means the global shape of a manifold. We shall also see how the above concepts are linked with the topology of a manifold.

The main goal of modern differential geometry is to study various geometric structures and constructions on a manifold. The goal of the first part of this course is to familiarize with the above "lingua franca"; the goal of the second part of this course is to get an idea on how to use analysis to study geometry.

Contents

1. Smooth manifold: immersion, submersion and embedding, tangent bundle and vector fields, differential of a map, cotangent bundle, tensor and differential forms, Lie derivatives, Frobenius theorem, Lie groups and matrix groups.

2. Riemannian metric: geodesics, Riemannian curvature, Levi-Civita connection, Gaussian normal coordinate, first and second variation, Jacobi field, examples to demonstrate how to calculate curvature and geodesics.

3. Classical results in Riemannian geometry: the Hopf-Rinow theorem, the Bonnet-Myers theorem, the Cartan-Hadamard theorem, space of constant sectional curvature.

4. Vector bundle: bundle constructions from linear algebra, connections and curvatures, the relation between principle bundles and vector bundles, characteristic classes in terms of curvature.

5. The de Rham/Hodge theory: de Rham cohomology, harmonic forms, Hodge decomposition of differential forms.

The lecturer shall cover the above materials in Differential Geometry (I, II), but needs not to do it in the above order. The above materials are supposed to occupy only 24$\sim$25 weeks out of 34 weeks. What follows are some possible topics for the rest of this course:

- The Atiyah-Singer index theorem.
- Minimal surface/submanifold.
- The Bochner-Weitzenböck technique: vanishing argument and/or the maximum principle (with eigenvalue estimates or harmonic maps).
- Some complex/Kähler geometry.
- Geometric flow.
- Others ...

Basically, the lecturer is suggested to use the rest of the course to do some topics closely related to modern researches, so that student can get some ideas about current development in differential geometry and about how to do research-oriented learning.

References

1. J. Cheeger and D.G. Ebin, *Comparison Theorems in Riemannian Geometry*, 1975.
2. M.P. do Carmo, *Riemannian Geometry*, 1992.
3. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry Vol. 1 & 2*, 1996.
4. P. Petersen, *Riemannian Geometry*, 2nd edition, 2006.
5. C. H. Taubes, *Differential geometry. Bundles, connections, metrics and curvature*, 2011.
6. F.W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, 1971.

Syllabuses for Honor Courses

Algebra I & II

Algebra is a fundamental part of the language of mathematics. Algebraic methods are used in all areas of mathematics. We will fully develop all the key concepts. We will also examine in more detail all the basic algebraic structures, various constructions in algebra. Level of abstraction increases as the full year course goes on. This course sets up a bridge between classical high-school algebra problems to the more advanced frontiers of modern mathematical researches. Additional topics in commutative algebra, or linear representations of finite groups will be covered if time is permitted.

Part 1. Group Theory.

Permutation groups, Cyclic groups, Abelian groups, Free groups, Group homomorphisms, Quotient groups, Group actions, Sylow theorems, Fundamental theorem of finite abelian groups, Direct product and semidirect product, Group extensions, Finite simple groups, Solvable groups.

Part 2. Ring Theory.

Rings, Ideals, Matrix rings, Integral domains, Rings of fractions, Euclidean domains, Principal ideal domains, Unique factorization domains, Rings of quadratic algebraic integers, Polynomial rings, Power series rings, Resultant, Hilbert basis theorem, Gröbner basis.

Part 3. Field Theory.

Field extensions, Fundamental theorem of Galois theory, Finite fields, Solvability by radicals, Cyclotomic extension, Kummer extension, Hilbert's theorem 90, Galois resolvent, Traces, Norms.

Part 4. Module Theory.

Tensor product, Free modules, Projective modules, Injective and flat modules, Symmetric and exterior algebras, Modules over PID, Introduction to Homological algebra.

Part 5. Additional Topics* (with emphasizes on examples).

*Introduction to Commutative Algebra:
Noetherian rings and Artinian rings, Associated primes and Primary decomposition, Going-up and Going-down theorems, Dedekind domains.

*Introduction to the representation theory of finite groups:
Linear representations, Characters, Induced characters, Induced representations.

Selected References:

M. Artin : Algebra.
Dummit-Foote: Abstract Algebra.
N. Jacobson: Basic Algebra I.
S. Lang: Undergraduate Algebra.
Van der Waerden: Algebra I, 7th edition.

Analysis I & II

(The First Semester)
Chapter 1. $\mathbb{R}^n$ and its topology (7 weeks)

§1.1 Introduction

Metric on $\mathbb{R}^n$, Schwartz inequality, limits, Cauchy sequence, series, completeness of $\mathbb{R}^n$, countable and uncountable sets. Examples.

§1.2 Topology of $\mathbb{R}^n$

1. Open set, interior of a set, closed set, closure, accumulation points, boundary. Examples.
2. Continuous functions (Examples, including monotone), uniform convergence and power series.

§1.3 Compact and connected sets. (1 week)

1. Compact sets, The Heine-Borel (Bolzano-Weierstrass) theorem and open covering.
2. Connected set, one-dimensional classification, path-connectedness, applications to continuous functions (Intermediate theorem). Examples. (§§1.1-1.3 for 3 weeks)

§1.4 Metric space

1. $\mathbb{R}^n$ (in ℓ^p)
2. The space of continuous functions, Ascoli-Arzelà theorem, Stone-Weierstrass theorems (compactness revisited).
3. Fixed point theorem.
   (a) Contraction maps.
   (b) Applications to O.D.E: existence and uniqueness, continuity on initial data, local stability in 2×2 system. (2 weeks)
   (c) Brouwer fixed points (optional).
4. Baire category theorem and its applications.

Chapter 2. Differentials (6 weeks)

§2.1

1. Overview (one-dimension) Rolle's theorem and mean-value theorem. Applications.
2. Linear transformation, Differential (definition) and example (with emphasis on geometry)
3. Partial derivatives and continuous P.D $\to$ differentials
4. The chain rule. The determinant of its differential (for $\mathbb{R}^n \to \mathbb{R}^n$)

§2.2 Higher order differentials. Partial derivatives. Taylor expansions.

§2.3 Implicit function theorem and its variations. Examples.

§2.4 Local extrema and Lagrange multipliers. Applications. Examples.

§2.5 Sard's theorem.

Chapter 3. Riemann Integral of multi-variables (1.5 weeks)

1. Definition, change of variable formulas (for continuous function). Examples.
2. Surface integral and line integral. Stokes' theorem. (Applications)
3. Lebesgue's criterion for the existence of Riemann integral. (1 week, the next semester)

(The Second Semester)

Chapter 4. Lebesgue integrals (8 weeks)

§4.1 Measure and measurable functions. (1 week)

§4.2 Lebesgue integrals. Discuss the difference between Lebesgue integrable and Riemann integrable functions. L^1 space.

§4.3 Convergence theorem:

1. Monotone, dominated convergence, and the Fatou lemma.
2. (Convergence almost everywhere)
3. Counterexamples (Weak convergence does not imply the convergence of the integral).
4. Convergence implies almost uniform convergence. (2 weeks for §§4.2, 4.3)

§4.4 The Fubini theorem and its applications. (1 week)

§4.5 Differentiation and integration. (4 weeks)

1. Hardy-littlewood maximal function, Vitali covering.
2. Lebesgue's point.
3. Differentiation under the integral.
4. Convolution.
5. Approximation to the identity.
6. BV functions.
7. Absolutely continuous function, etc.

Chapter 5. Fourier theory (Introduction) (5 weeks)

1. $L^2[0,1]$, Fourier series, Parseval's theorem, and the completeness, examples.
2. $L^2(\mathbb{R})$, Fourier transformation and the inverse Fourier transformation, the Parseval theorem and the Poisson summation, examples.
3. Applications to Laplace equation, Heat equation, and hyperbolic equation.

Complex Analysis

Complex function theory is a valuable tool used in many branches of pure, applied mathematics and natural sciences, including geometry, number theory, ordinary differential equations, partial differential equations and various topics in physics and engineering. A basic course shall enable student to understand the concept of complex analyticity, to use residue calculus for evaluation of integrals, to understand conformal mappings, and to learn some additional topics selected from Riemann mapping theorem, special/elliptic functions, prime number theorem, complex dynamical systems, etc.

Contents: (those with * are optional)

Chapter 1: Cauchy theory (4 weeks)

Cauchy-Goursat theorem and Cauchy's integral formula, singularities of analytic functions and Laurent series, residue theorem and its applications: maximum modulus principle, open mapping theorem, argument principle, Rouche's theorem and the evaluation of integrals.

Chapter 2: Entire functions (4 weeks)

Jensens's formula, Weierstrass infinite products, Hadamard's factorization theorem, analytic continuation of gamma and zeta functions, prime number theorem.

Chapter 3: Conformal mappings (4 weeks)

Conformal mapping on basic domains and Schwarz Lemma, hyperbolic geometry Riemann mapping theorem, Schwartz reflections and boundary behavior Schwartz-Christoffel integrals, harmonic functions and the Dirichlet problem for multiply-connected regions.

Chapter 4: Special topics-select two from the following: (4 weeks)

*Compact family of meromorphic functions, theorem of Montel and Picard, applications

*Introduction to complex dynamical systems, Julia sets,

*Introduction to elliptic functions and modular functions,

*Introduction to linear ODE, monodromy groups, and analytic continuations.

Selected references:

1. Stein, E.M., Shakarchi, R., "Complex analysis"
2. Lang, S., "Complex analysis", GTM 103
3. Ahlfors, L., "Complex analysis"
4. Whittaker-Watson, "Modern Analysis"
5. Gamelin, T.W., "Complex Analysis"

Geometry

Modern geometry is widely used in pure and applied mathematics as well as in physics and engineering. This course starts with the classical theory of curves and surfaces in three dimensional Euclidean spaces, which provides the most visualizable and fundamental examples for applications as well as for higher dimensional theories. It then moves to the foundation of differentiable manifolds and vector bundles. The aim of this course is to provide sufficient background for future study on Riemannian geometry, symplectic geometry, differential topology, as well as global analysis (PDE, ODE) on manifolds.

Syllabus (those with * are optional)

1. Theory of surfaces (6 weeks)

1.1 Plane curves and space curves, Frenet frame

1.2 First and second fundamental forms of surfaces in Euclidean spaces

1.3 Abstract surfaces, Levi-Civita connection and parallel translations

1.4 Gaussian curvature, holomomy angle, Gauss-Bonnet theorem and its applications

1.5 Gauss-Codazzi equations, mean curvature, minimal surfaces, and *the Weierstrass representations

2. Differentiable manifolds (5 weeks)

2.1 Definitions and examples of differentiable manifolds and differentiable maps

2.2 Tangent bundles, tangent maps, and Sard's theorem

2.3 Embeddings, immersions, submersions, and Whitney's theorem

2.4 Flow of a vector field, Lie derivatives, and Frobenius' theorem

2.5 Introduction to Lie groups and Lie algebras through examples (classical groups)

3. Differential forms and vector bundles (5 weeks)

3.1 Differential forms and Stokes' theorem

3.2 De Rham cohomology, Poincare duality, and de Rham theorem

3.3 Connections and curvature on vector bundles

3.4* Chern-Weil theory of characteristic classes

3.5* Flat bundles and representations of the fundamental group

Bibliography

D. Struik: Lectures on classical differential geometry

M. Do Carmo: Differential geometry of curves and surfaces

M. Hirsch: Differential topology

M. Spivak: A comprehensive introduction to differential geometry vol I and II

J. Milnor: Characteristic classes

R. Bott and L. Tu: Differential forms in algebraic topology