Workshop on Recent Developments in Evolutionary Equations and Related Topics

Date: 20 November, 2023

9:40 -10:00 **Registration**

10:00-10:10 **Opening**

10:10-10:50 **Shoji Yotsutani** (Ryukoku University)
All global bifurcation curves of stationary solutions to a phase field model

10:50-11:10 **Coffee Break**

11:10-11:50 **Jenn-Nan Wang** (National Taiwan University)
Second order elliptic equations with discontinuous complex coefficients: quantitative uniqueness estimates and size estimate

11:50-13:30 **Lunch Break**

13:30-14:10 **Yoshitsugu Kabeya** (Osaka Metropolitan University)
Structure of positive solutions to a Choquard-type elliptic system with a potential term

14:20-15:00 **Ryo Takada** (University of Tokyo)
Large time behavior of global solutions to the rotating Navier-Stokes equations

15:00-15:20 **Coffee Break**

15:20-16:00 **Jann-Long Chern** (National Taiwan Normal University)
Existence of weak solutions to a general porous medium equation with nonlocal pressure

16:10-16:50 **Kazuhiro Ishige** (University of Tokyo)
Existence of solutions to a fractional semilinear heat equation in uniformly local weak Zygmund type spaces

18:20 Workshop Banquet

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10:20-11:00 **Van Tien Nguyen** (National Taiwan University)

11:00-11:20 **Coffee Break**

11:20-12:00 **Eiji Yanagida** (University of Tokyo)

Shoji Yotsutani (Ryukoku University)

Title: All global bifurcation curves of stationary solutions to a phase field model

Abstract: We are concerned with bifurcation curves of stationary solutions to a phase field model proposed by Fix and followed by Caginalp. We show all the global bifurcation curves of stationary solutions to the model in the 1-dimension case, and investigate the stability of them. We see that bifurcation curves are surprisingly rich in variety depending on the latent heat and the initial total enthalpy. For instance, bifurcation diagrams include the secondary bifurcation point where symmetric breaking occurs, and curves which connect a limit of boundary layer solutions to the other limit of internal layer solutions. This talk is based on recent joint works with Y. Miyamoto (Univ. Tokyo), T. Mori (Misashino Univ.), S. Tasaki (Hokkaido Univ.) and T. Tsujikawa (Miyazaki Univ.).

Jenn-Nan Wang (National Taiwan University)

Title: Second order elliptic equations with discontinuous complex coefficients:
quantitative uniqueness estimates and size estimate

Abstract: In this talk, I would like to derive three-ball inequalities and propagation of smallness for the complex second order elliptic equation with discontinuous Lipschitz coefficients. Second order elliptic equations with complex coefficients arise from some well-known physical models such as the biological tissues or the electromagnetic waves propagating in a conductive medium. As an application of such estimates, we study the size estimate problem by one pair of Cauchy data on the boundary, that is, a pair of the Neumann and Dirichlet data of the solution on the boundary. The main ingredient in the derivation of three-ball inequalities and propagation of smallness is a local Carleman estimate proved recently.

Yoshitsugu Kabeya (Osaka Metropolitan University)

Title: Structure of positive solutions to a Choquard-type elliptic system with a potential term

Abstract:

We consider the following system of ordinary differential equations

$$(1)\qquad \begin{cases} -r^{1-d}\big(r^{d-1}u'\big)' - V(r)u &= \phi|u|^{p-2}u, \qquad u(0)=\alpha, \\ -r^{1-d}\big(r^{d-1}\phi'\big)' &= |u|^{p}, \qquad\quad \phi(0)=\sigma, \end{cases}$$

where $d \geq 3$ is a natural number, $\alpha, \sigma > 0$ are free parameters and $p > (d+2)/(d-2)$.

The potential $V \in C^1((0,\infty))$ is assumed to satisfy the assumptions

$$(V)\quad \begin{cases} \text{(i) } rV(r) \in L^1((0,1)), \\ \text{(ii) } V(r) \geq 0,\ V(r) \not\equiv 0, \\ \text{(iii) } V(r) = \dfrac{\omega}{r^2} + \dfrac{\omega_1}{r^{2+\delta}} + V_1(r) \text{ near } r=\infty, \end{cases}$$

where $0 \leq \omega < C_H := (d-2)^2/4$, $\omega_1 \neq 0$, $\delta > 0$ and

$$\lim_{r\to\infty} r^{2+\delta+\varepsilon}V_1(r) = \lim_{r\to\infty} r^{3+\delta+\varepsilon}V_1'(r) = 0,$$

for some $\varepsilon > 0$.

We will show the structure of positive solutions to (1) for suitable ranges of p and δ. This talk is based on the joint work with Professor Vitaly Moroz (Swansea University, UK) and Dr. Damiano Greco (Swansea University, UK).

Ryo Takada (University of Tokyo)

Title: Large time behavior of global solutions to the rotating Navier-Stokes equations

Abstract: We consider the large time behavior of global solutions for the initial value problem of the Navier-Stokes equations with the Coriolis force in the three-dimensional whole space. We

establish the L^p temporal decay estimates with the dispersion effect of the Coriolis force for global solutions. Moreover, we show the large time asymptotics of global solutions behaving like the first-order spatial derivatives of the integral kernel of the corresponding linear solution. This talk is based on the joint work with Takanari Egashira (Kyushu University).

Jann-Long Chern (National Taiwan Normal University)

Title: Existence of weak solutions to a general porous medium equation with nonlocal pressure

Abstract: In this presentation, we will delve into the regularity and symmetry characteristics of solutions within the context of a porous media equation. The regularity of these solutions was thoroughly investigated by Caffarelli, Gualdani, and Zamponi in their 2020 study. This field of research has presented considerable challenges and remains an ongoing area of inquiry. Furthermore, by employing time-discretization techniques, we illustrate that each discretized solution corresponds to a solution of an elliptic-type equation. We extend the methodology initially introduced by Caffarelli and Silvestre, and introduce novel corollaries to establish the existence of weak solutions for a wide range of porous medium equations incorporating nonlocal effects through a time-discretization approach.

Kazuhiro Ishige (University of Tokyo)

Title: Existence of solutions to a fractional semilinear heat equation in uniformly local weak Zygmund type spaces

Abstract: We introduce uniformly local weak Zygmund type spaces, and obtain an optimal sufficient condition for the existence of solutions to the critical fractional semilinear heat equation. This is a joint work with Tatsuki Kawakami (Ryukoku Univ.) and Norisuke Ioku (Tohoku Univ.)

Van Tien Nguyen (National Taiwan University)

Title: Blowup patterns for the Keller-Segel system in dimensions 3 and 4

Abstract The Keller-Segel system is known to exhibit rich dynamical behaviors including singularity formation with self-similarity structure. The talk presents recent developments in the study of blowup for this system, and provides a rigorous construction leading to one of the blowup solutions which was formally discussed by Brenner-Constantin-Kadanoff-Schenkel-Venkataramani in [Nonlinearity 1999].

Eiji Yanagida (University of Tokyo)

Title: Traveling singular solutions of the fast diffusion equation

Abstract: In this talk, we consider traveling singular solutions of the fast diffusion equation. By assuming a special form of traveling solutions, the problem can be reduced to an ODE on a finite interval. We classify solutions depending on the shape of singular sets, and study the existence of traveling singular solutions of various types. It is shown that the structure of traveling singular solutions depends on a parameter, and there appear some critical exponents depending on the spatial dimension. The results are applied to show the existence of more general solutions with dynamic singularities.