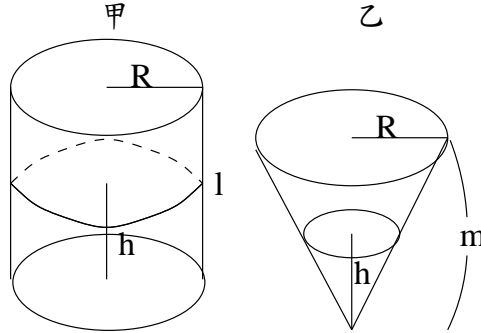


有甲乙兩個水漏容器：

甲是一個高為 $l$ ，底是半徑為 $R$ 的圓柱形。

乙是一個高為 $m$ ，底是半徑為 $R$ 的圓錐體。



在底部各有一個半徑為 $r$ 的小孔，已知裝滿水的甲乙兩容器水同時漏光，請問 $l$ 和 $m$ 的關係為何？ $V_1$ ：甲剩下水的體積

$V_2$ ：乙剩下水的體積

$$\frac{dV_1}{dt} = \frac{dV_1}{dh} \cdot \frac{dh}{dt} = -\pi r^2 \sqrt{2gh}, \text{ by 托里切利定律}$$

$$\frac{dV_1}{dh} = \pi R^2$$

$$\Rightarrow \pi R^2 \frac{dh}{dt} = -\pi r^2 \sqrt{2gh}$$

$$\Rightarrow \frac{dh}{\sqrt{h}} = -\frac{r^2}{R^2} \sqrt{2g} dt$$

$$\Rightarrow 2h^{\frac{1}{2}} = -\frac{r^2}{R^2} \sqrt{2g} t + c$$

$$\text{當 } t = 0, h = l, \Rightarrow c = 2l^{\frac{1}{2}}$$

$$\therefore \text{甲漏光水所須時間} = \frac{2l^{\frac{1}{2}}}{\frac{r^2}{R^2} \sqrt{2g}}$$

$$\frac{dV_2}{dt} = \frac{dV_2}{dh} \cdot \frac{dh}{dt} = -\pi r^2 \sqrt{2gh}$$

$$\frac{V_2}{\frac{1}{3}\pi R^2 m} = \frac{h^3}{m^3}$$

$$\therefore V_2 = \frac{\pi R^2 h^3}{3m^2}$$

$$\Rightarrow \frac{dV_2}{dh} = \frac{\pi R^2 h^2}{m^2}$$

$$\therefore \frac{\pi R^2 h^2}{m^2} \cdot \frac{dh}{dt} = -\pi r^2 \sqrt{2gh}$$

$$h^{\frac{3}{2}} dh = -\frac{m^2 r^2 \sqrt{2g}}{R^2} dt$$

$$\frac{2}{5} h^{\frac{5}{2}} = -\frac{m^2 r^2 \sqrt{2g}}{R^2} t + c$$

$$\text{當 } t = 0, h = m, \Rightarrow c = \frac{2}{5} m^{\frac{5}{2}}$$

$$\therefore \text{乙漏光水所須時間} \frac{\frac{2}{5} m^{\frac{5}{2}}}{\frac{m^2 r^2 \sqrt{2g}}{R^2}} \therefore \text{甲乙同時漏光} \therefore \frac{2l^{\frac{1}{2}}}{\frac{r^2}{R^2} \sqrt{2g}} = \frac{\frac{2}{5} m^{\frac{5}{2}}}{\frac{m^2 r^2 \sqrt{2g}}{R^2}}$$

$$\Rightarrow \sqrt{m} = 5\sqrt{l}$$

$$\Rightarrow m = 25l$$