Algebraic Surfaces

Homework 1

We work on a non-singular projective variety X.

(1) Show that $D_1 + D_2$ is ample if both D_1, D_2 are ample.

(2) Show that for any divisor D and an ample divisor A, there is $n > 0$ such that $D + nA$ is ample.

(One might consider divisors with $\mathbb{Q}$ coefficient, then this is equivalent to $A + \epsilon D$ is ample. So ampleness is an open condition).

(3) We say a divisor D is *nef (= numerically effective= numerically eventually free)* if $D.C \geq 0$ for all irreducible curve C. Show that if D is nef and A is ample, then $D + A$ is ample.

(4) Let D be a nef divisor on a surface X. Suppose that $D.D > 0$. Prove that $h^0(X, nD) \neq 0$ for some $n \gg 0$. And prove that $h^0(X, nD) \geq \frac{1}{2}D^2n^2 + o(n)$ as $n \to \infty$