# 數學有何用

陳宜良
應用數學講座教授
中央大學數學系
2014.10.28

# 數學有何用

- 數學是追求樣式規律性與本質性的學科
- 數學是理性思維的工具

  - 探索未知
  - 解決現實問題
  - 開創未來世界

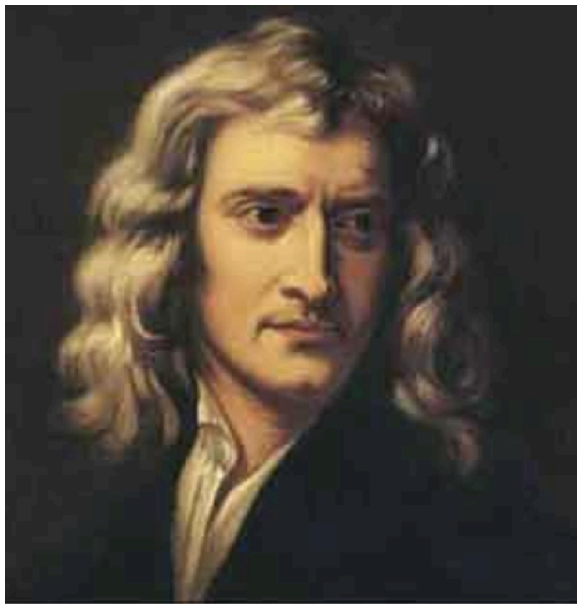
需要數學

# 一個探索未知的例子

- **Tycho Brahe (第谷·布拉赫1546-1601)**
  - 一組改變歷史的數據
- **Johannes Kepler (刻卜勒1571-1630)**
  - 一個追求真理的探索
- **Isaac Newton(牛頓1642-1727)**
  - 千古謎題破解日,萬有引力發現時

# 探索未知

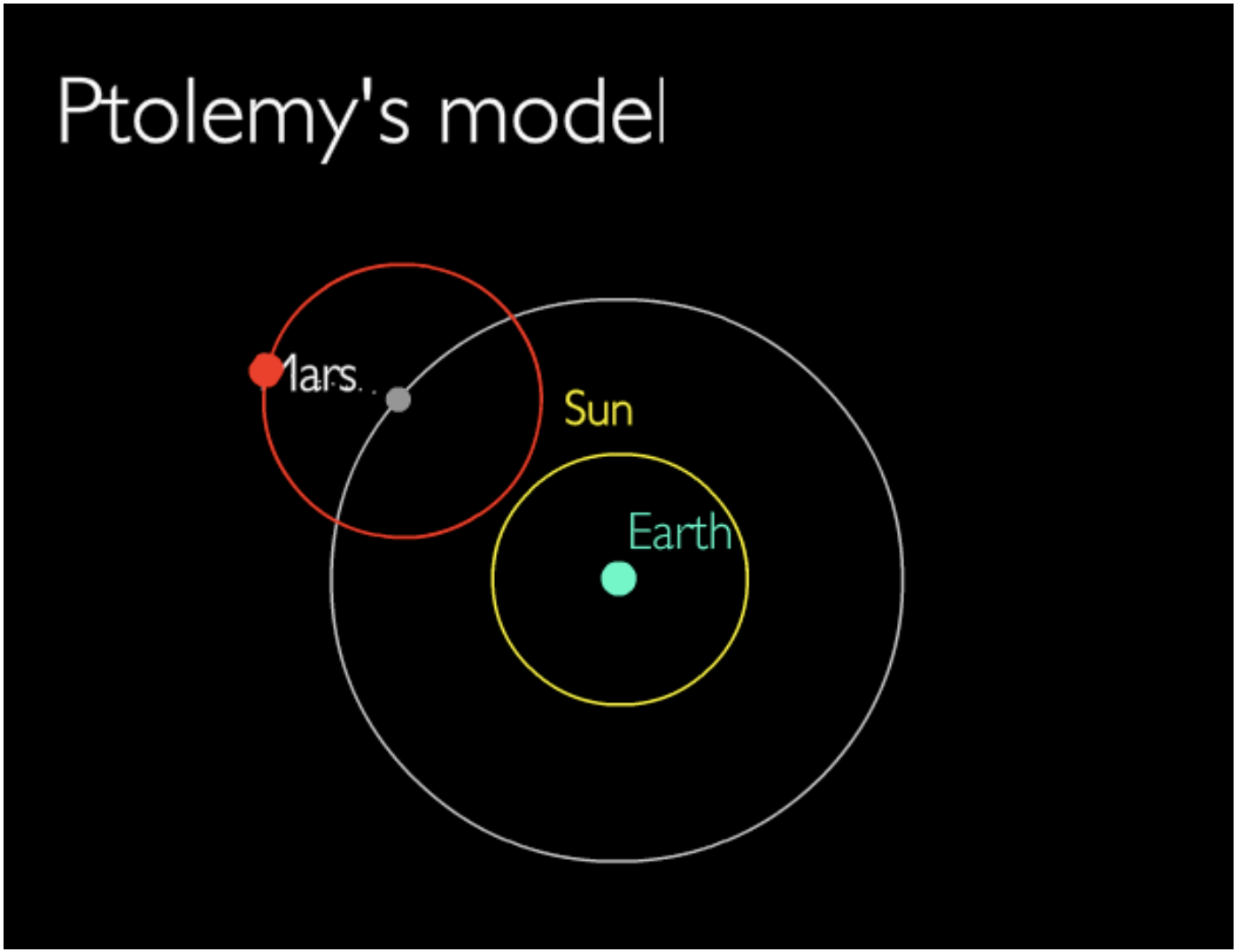
- 歷史上有名的案例



# 探索未知

# 探索宇宙的奥秘

# 托勒密模型



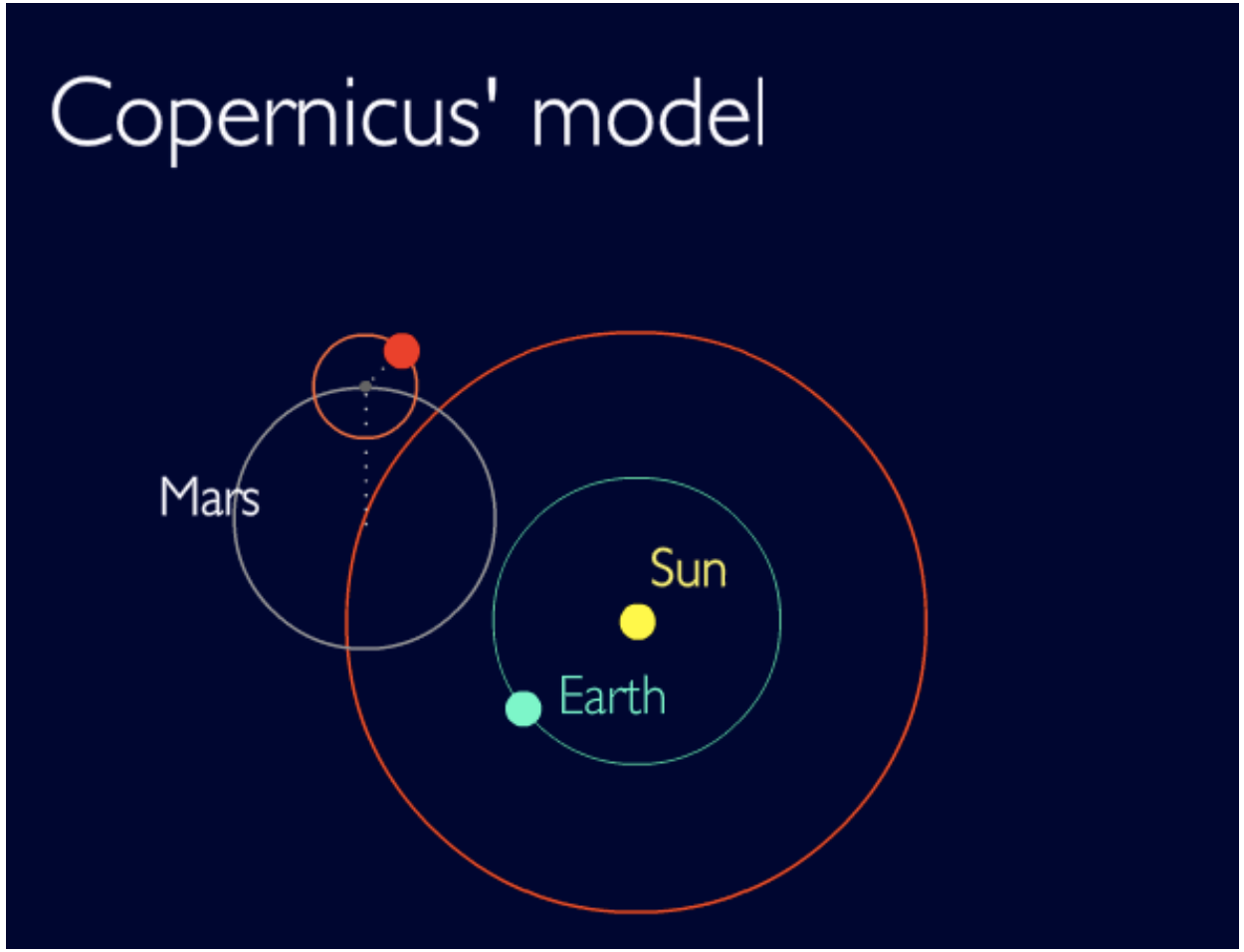
# Geocentric Model (地心説)

- In the 4th century BC, two influential Greek philosophers, Plato and his student Aristotle, wrote works based on the geocentric model. According to Plato, the Earth was a sphere, stationary at the center of the universe. The stars and planets were carried around the Earth on spheres or circles, arranged in the order (outwards from the center): Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, fixed stars, with the fixed stars located on the celestial sphere.

- In the Ptolemaic system, each planet is moved by a system of two spheres:
  one called its deferent, the others, its epicycle
- The deferent is a circle whose center point, called the eccentric and marked in the diagram with an X, is removed from the Earth. The original purpose of the eccentric was to account for the differences of the lengths of the seasons (autumn is the shortest by a week or so), by placing the Earth away from the center of rotation of the rest of the universe.

- Another sphere, the **epicycle(周轉圓)**, is embedded inside the deferent sphere and is represented by the smaller dotted line to the right. A given planet then moves around the epicycle at the same time the epicycle moves along the path marked by the deferent

# 哥白尼模型



Tycho Brahe was a Danish nobleman known for his accurate and comprehensive astronomical and planetary observations.

http://en.wikipedia.org/wiki/Tycho_Brahe

- Tycho realized that progress in astronomy required systematic, rigorous observation, night after night, using the most accurate instruments obtainable. This program became his life's work. Tycho improved and enlarged existing instruments, and built entirely new ones.

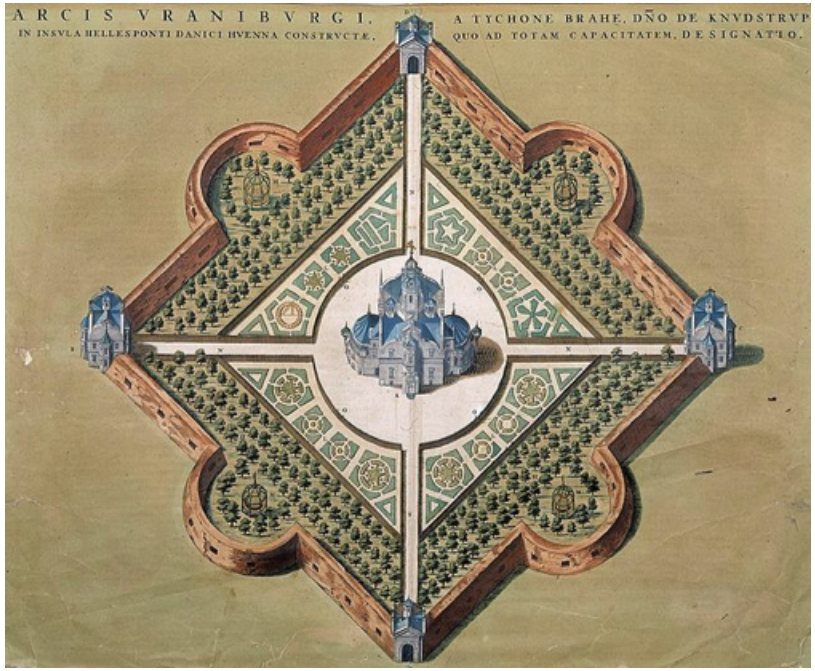
- The king of Denmark gave Tycho tremendous resources: an island with many families on it, and money to build an observatory. (One estimate is that this was 10% of the gross national product at the time!)

- Tycho Brahe was granted an estate on the island of Hven and the funding to build the Uraniborg, an early research institute, where he built large astronomical instruments and took many careful measurements, and later Stjerneborg, underground, when he discovered that his instruments in the former were not sufficiently steady.

- The observatory had a large mural quadrant affixed to a north-south wall, used to measure the altitude of stars as they passed the meridian. This, along with many other instruments of the observatory, was depicted and described in detail in Brahe's 1598 book *Astronomiae instauratae mechanica*.

# Accuracy

- He aspired to a level of accuracy in his estimated positions of celestial bodies of being consistently within 1 arcminute of their real celestial locations, and also claimed to have achieved this level. But in fact many of the stellar positions in his star catalogues were less accurate than that. The median errors for the stellar positions in his final published catalog were about 1'.5, indicating that only half of the entries were more accurate than that, with an overall mean error in each coordinate of around 2'.

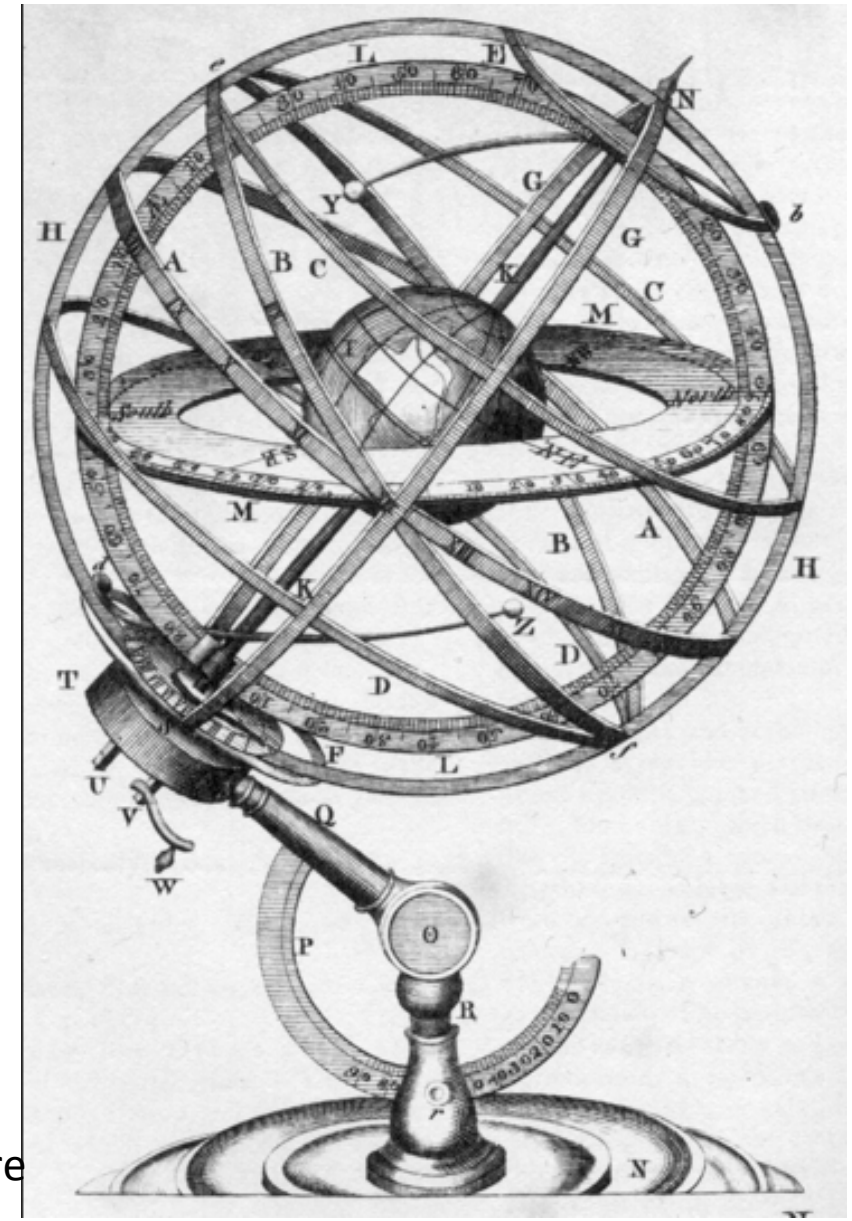

ARCIS VRANIBVRGI.
IN INSVLA HELLESPONTI DANICI HVENNA CONSTRVCTÆ.
A TYCHONE BRAHE, DÑO DE KNVDSTRVP
QVO AD TOTAM CAPACITATEM, DESIGNATÆ.


ORTHOGRAPHIA PRÆCIPVÆ DOMVS ARCIS VRANIBVRGI

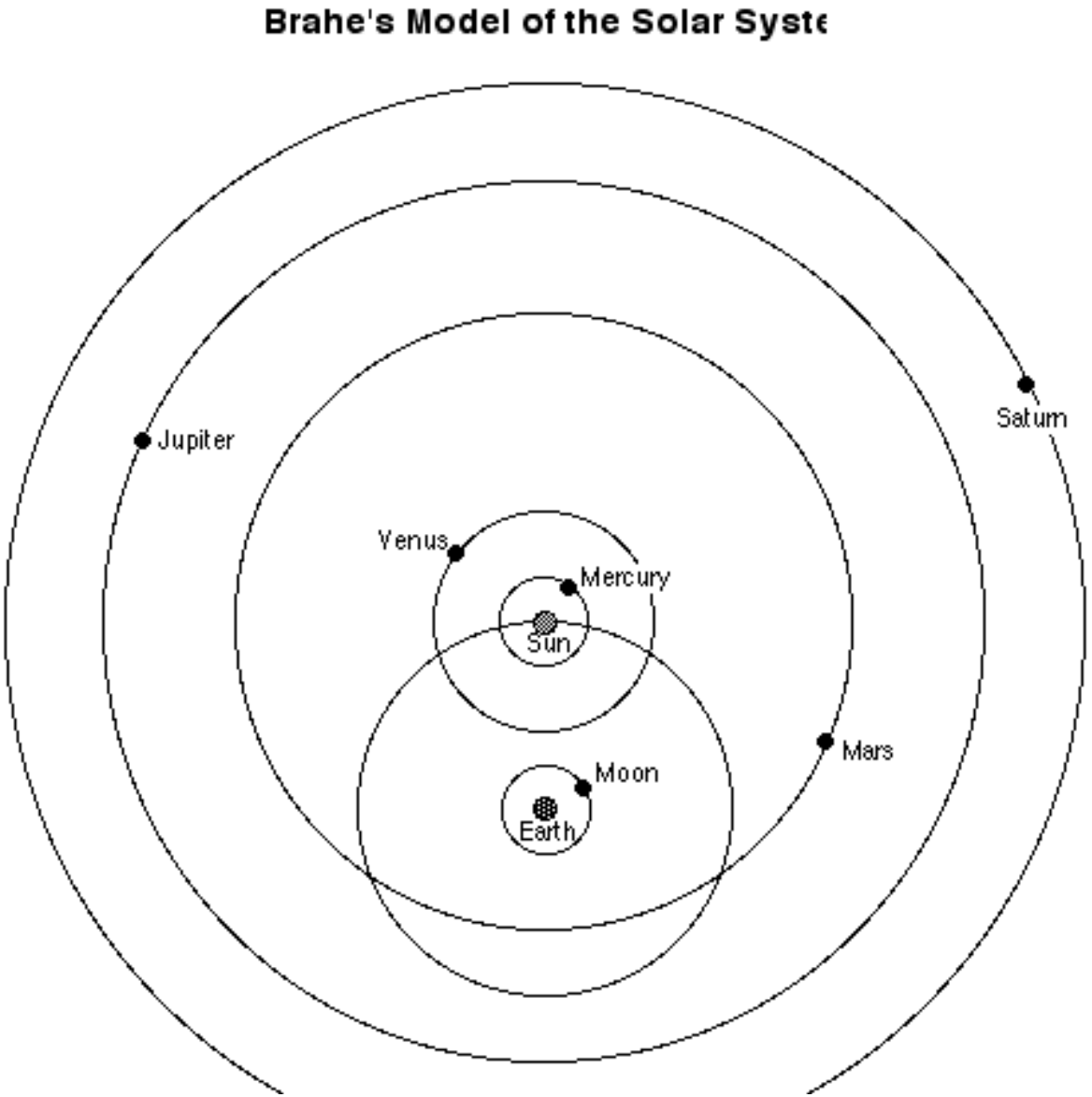
- After disagreements with the new Danish king Christian IV in 1597, he was invited by the Bohemian king and Holy Roman emperor Rudolph II to Prague, where he became the official imperial astronomer. He built the new observatory at Benátky nad Jizerou. There, from 1600 until his death in 1601, he was assisted by Johannes Kepler who later used Tycho's astronomical data to develop his three laws of planetary motion.

Eduard Ender's portrait of 1855 showing Tycho demonstrating a celestial globe to the Emperor Rudolph II in Prague.

http://en.wikipedia.org/wiki/Armillary_sphere



# Tycho’s model of solar system



# 刻卜勒的探索

- http://www.keplersdiscovery.com/
- 刻卜勒簡介
- 刻卜勒的新天文學

- 從刻卜勒到牛頓 --千古謎題破解日,萬有引力發現時 項武義・張海潮
- 牛頓萬有引力

Isaac Newton（1642～1727），英國的數學家及物理學家，微積分主要締造者，萬有引力理論的發明者。

- Tycho's observations were accurate enough for Kepler to discover that the planets moved in elliptic orbits, and his other laws, which gave Newton the clues he needed to establish universal inverse-square gravitation.

這是做專題的好題材。

# 解決現實生活中的問題

## 數學與真實世界連結

- 可以讓我們理性分析生活周遭所發生的事
- 可以防災
- 可以讓生活便利
- 可以增進效率
- 可以理性的判斷與協助決策
- 可以協助解決問題

# 第一類的例子

- 柯南: 死亡時間的推斷
- 馬爾薩斯: 人口成長的趨勢

# 柯南如何判斷受害者死亡時間

- **一屍體在**6:30 AM被發現，屍體溫度為 20◦. 7:30 AM時，屍體溫度為18◦. 當時室溫為16◦ 。人體溫約為37◦. 柯南怎麼估計受害者死亡時間呢？

# 數學建模

- 設屍體溫度在時刻 t 時為Y(t)
- 室溫為 K
- 模型:

$$\frac{Y(t+\Delta t)-Y(t)}{\Delta t}=\alpha\left(K-Y(t)\right)$$

- 由Y 在兩個時間點的值推斷$\alpha$與初期始時間

# Euler 方法

**Leonhard Euler**
1707-1783

- Initial: $Y(0)$
- Repeat: for $n = 1, , ..., N$

$$Y(n\Delta t) = (1 + \alpha\Delta t)Y((n-1)\Delta t) - \alpha\Delta t K.$$

- Output: $Y(N\Delta t)$.

由Y 在兩個時間點的值推斷 $\alpha$與初期始時間

# 馬爾薩斯(Malthus)人口論

- 馬爾薩斯Malthus人口論指出人口呈幾何成長，糧食受限於耕地面積的限制，終究會達到飽和，而有限的土地分配給呈幾何成長的人口，終究不足分配，若土地可以買賣，則其價值也會愈來愈高。

- 這個理論也可以解釋貴族體制崩解的原因。

AN

# ESSAY

ON THE

*PRINCIPLE OF POPULATION,*

AS IT AFFECTS

THE FUTURE IMPROVEMENT OF SOCIETY.

WITH REMARKS

ON THE SPECULATIONS OF MR. GODWIN,

M. CONDORCET,

AND OTHER WRITERS.

---

LONDON:

PRINTED FOR J. JOHNSON, IN ST. PAUL'S CHURCH-YARD.

1798.

Thomas Robert Malthus
(1766-1834)

# 人口論的數學模型(Malthus, 1798)

$$P(t+\Delta t) = P(t)(1+r\Delta t)$$

$$P(n\Delta t) = P(0)(1+r\Delta t)^n$$

- 人類社會的人口若不作適當控制，終會被細菌或戰爭所摧毀。

# 環境承載有上限

Pierre Verhulst (1804-1849

- 環境承載量 $R := r/\beta$

$$\frac{P(t+\Delta t) - P(t)}{\Delta t} = rP(t) - \beta P(t)^2$$

$$= rP(t)\left(1 - \frac{P(t)}{R}\right)$$

$$\frac{\Delta P(t)}{\Delta t} = rP(t)\left(1 - \frac{P(t)}{R}\right)$$

# 數值方法: 尤拉法

- 初始值 $P(0)$
- 迭代公式: $\text{for } n = 0, \ldots, N-1$

$$P((n+1)\Delta t) = (1 + r\Delta t)P(n\Delta t)\left(1 - \frac{P(n\Delta t)}{R}\right)$$

- 終點值 $P(N\Delta t)$

$$P(t) \to R \text{ as } t \to \infty.$$

# 漁業 養殖業: 永續經營

- 設每單位時間漁獲量為 C

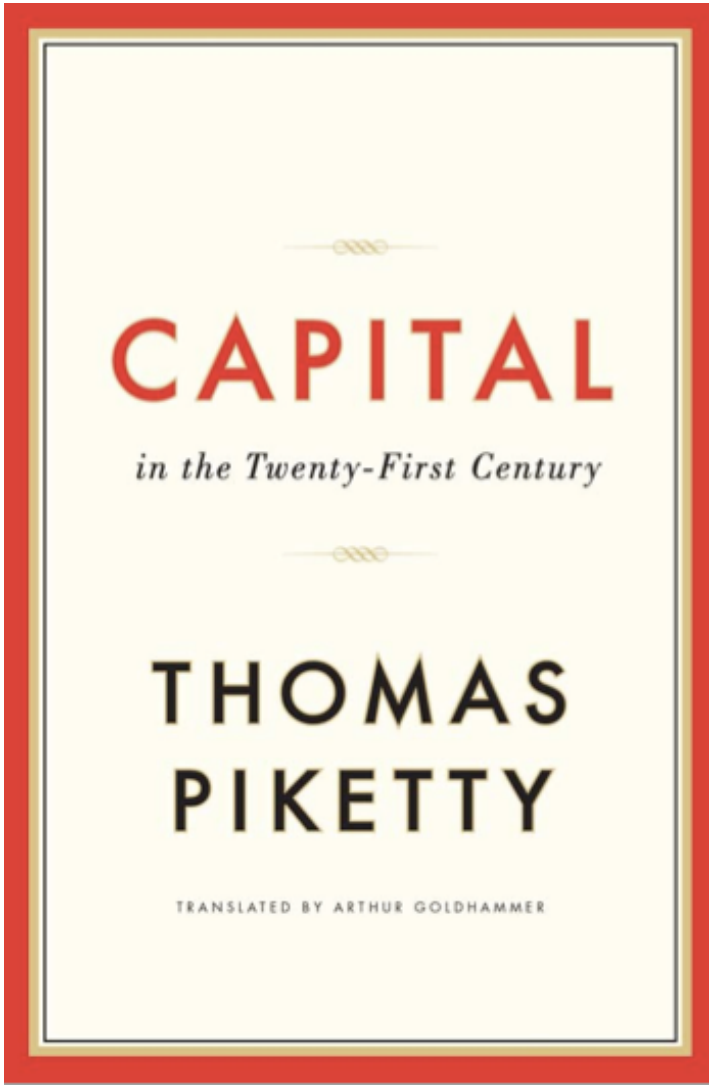
$$\frac{\Delta P(t)}{\Delta t} = rP(t)\left(1 - \frac{P(t)}{R}\right) - C$$

- 問題: C 應選擇何值時可以永續經營?

# 永續的社會

- 數學建模可以讓我們理性分析生活周遭所發生的事。
  - 人口成長
  - 糧食、水的需求
  - 都市擴張
  - 房價高漲

- 法國經濟學家Thomas Piketty 近日出了一本有名的書: 廿一世紀資本論




書中有許多數據值得數學建模與並做更進一步的分析。

# 第二類例子

- 亞瑪遜書局如何推薦書
- 影視租賃公司如何推薦影帶
- 歐巴馬團隊如何估計社群政黨取向
- 如何從DNA資料判斷疾病

# 亞馬遜如何推薦書

- 亞馬遜網上書店 Amazon.com 是美國最大的一家網路電子商務公司，是網路上最早開始經營電子商務的公司之一，亞馬遜書店成立於1995年，一開始只經營網路的書籍銷售業務，現在則擴及了範圍相當廣的其他產品，包括了DVD、音樂光碟、電腦、軟體、電視遊戲、電子產品、衣服、傢具等等。
- 亞馬遜會根據你購買或評比過的書籍，推薦你相關的書籍

# Collaborative Filtering

- Given (incomplete) data of rating
- Determine whether Albert will like or dislike item 5

| | Item1 | Item2 | Item3 | Item4 | Item5 |
|---|---|---|---|---|---|
| Albert | 3 | 4 | 5 | 3 | ? |
| User1 | 5 | 2 | 4 | 4 | 3 |
| User2 | 3 | 2 | 2 | 1 | 5 |
| User3 | 2 | 5 | 5 | 3 | 4 |
| User4 | 4 | 1 | 3 | 4 | 2 |

# Rating by correlation-weights

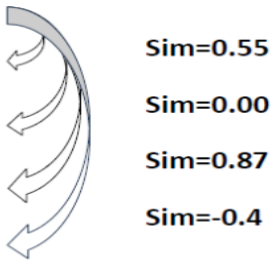
- Define similarity between two user $a$, $b$ by the correlation

$$\text{sim}(a,b) = \frac{\sum_p (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_p (r_{a,p} - \bar{r}_a)^2}\sqrt{\sum_p (r_{b,p} - \bar{r}_b)^2}}$$

- Define prediction of rating of user $a$ on item $p$ by

$$\text{pred}(a,p) = \bar{r}_a + \frac{\sum_{b \in N} \text{sim}(a,b) * (r_{b,p} - \bar{r}_b)}{\sum_{b \in N} \text{sim}(a,b)}$$

| | Item1 | Item2 | Item3 | Item4 | Item5 |
|---|---|---|---|---|---|
| Albert | 3 | 4 | 5 | 3 | ? |
| User1 | 5 | 2 | 4 | 4 | 3 |
| User2 | 3 | 2 | 2 | 1 | 5 |
| User3 | 2 | 5 | 5 | 3 | 4 |
| User4 | 4 | 1 | 3 | 4 | 2 |




Reproduce from Powerpoint-Slides for Recommender Systems - An Introduction

# Netflix Problem


- Netflix是一家美國公司，提供北美地區綫上播放DVD的出租業務。
- 2011年4月，Netflix宣布在美有2360萬用戶，而全世界則超過2600萬，可提供10萬種DVD選擇。
- Netflix會根據用戶訂閱過或評比過的資料提供推薦影片 。

# Netflix獎

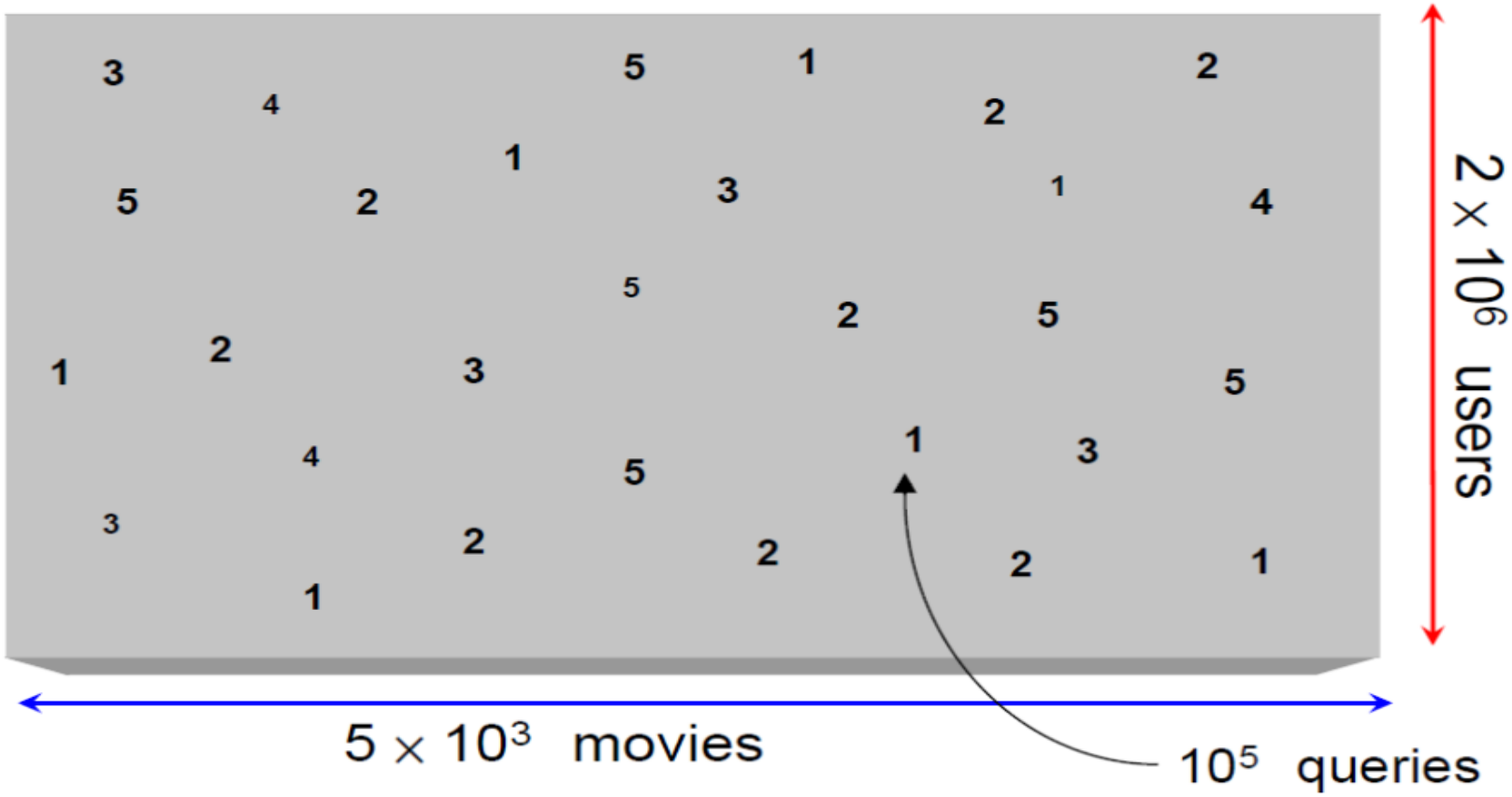
- Netflix為追求最佳collaborative filtering算法的獎，獎金一百萬美金，由Netflix公司提供。
- 比賽方式是所提供的方法要比該公司用戶的方法(Cinematch)好10%. 度量方法是RMSE (root-mean-square-error).

# Netflix問題

- Data format: <user, movie, date of grade, grade>
- Training data: Training set (99,072,112 ratings not including the probe set, 100,480,507 including the probe set)
- Quiz set (1,408,342 ratings), used to calculate leaderboard scores
- Test set (1,408,342 ratings), used to calculate competition scores
- On September 18, 2009, Netflix announced team "BellKor's Pragmatic Chaos" as the prize winner (a Test RMSE of 0.8567)

# A Matrix Completion Approach

- Given less than 1% of movie ratings
- Goal: predict missing ratings



# 第三類例子

- Google如何搜尋你要找的資料

# How google works

- 爬挖資料 (Crawling)
- 建立索引 (Indexing) 60 trillion pages
- 排序(Search algorithm and page ranking)

# How google works

Three steps to make search engine work [1]

- Googlebot: Google's web crawling robot
  It crawls, downloads and fetches webpages in the cyberworld to the indexer.
- Indexer: sorts and indexes every word on every page, and stores them in a huge database
- Query processor: perform page ranking algorithm upon your query

1. Lars Elden, Matrix Methods in Data Mining and Pattern Recognition, SIAM
2. PageRank, Wiki
3. GoogleGuide, how google works

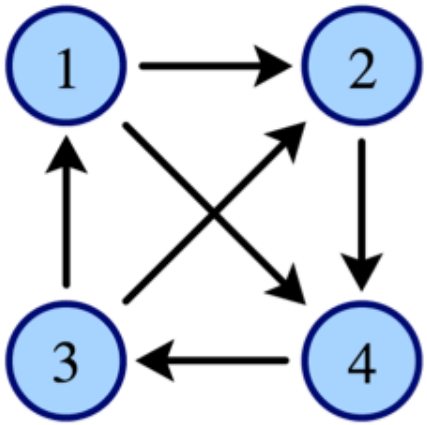
# Query Processor--PageRank

- It is a link analysis of webpages
- It was first suggested by Gabriel Pinski and Francis Narin in 1976 for ranking scientific journals.
- Two Stanford graduate students Sergey Brin and Larry Page developed PageRank as a new kind of search engine in 1996. They founded Google Inc. in 1998. PageRank is the basis of google's search tools.

# PageRank: A link analysis

- Each page (with inquiry words) a node of a graph and is ordered from $1, ..., n$
- $N_j$: the number of pages that $j$ is linked to (out-link).
- Define $Q_{ij} = 1/N_j$ if there is a link from $j$ to $i$.

$$\sum_i Q_{ij} = 1.$$




$$Q = \begin{bmatrix} 0 & 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1 & 0 & 0 \end{bmatrix}$$

- Weight $r_i \geq 0$: the importance of page $i$ to the inquiry.
- Normalized by $\sum_j r_j = 1$.
- The importance $r_i$ is weighted by

$$r_i = \sum_j \frac{r_j}{N_j} = \sum_j Q_{ij} r_j.$$

- It is an eigenvalue problem!

$$\boxed{Qr = r.}$$

# A random walk interpretation

- $(r_1, \cdots, r_n)^T$ is a distribution of the importance of each page for a particular inquiry.
- Start from, say $r^{(0)} = (\frac{1}{n}, \cdots, \frac{1}{n})^T$, perform $Qr^0$.
- We can continue this process

$$r^{(n+1)} = Qr^{(n)},$$

  This is called a random walk.
- Hopefully $r^{(n)}$ converges to $r$ with

$$Qr = r.$$

  If so, then the ranking is determined by the order of magnitudes of $r_i$.

# Two problems

- We may get stuck at some page
- The random walk may not converge

- If column $j$ is zero, which means that $j$ does not link to any other page. To avoid the random walk gets stuck at page $j$, we modify $Q$ to $P$ by: if the $j$th column is zero, replace this column by $e/N$, where $e := (1, \cdots, 1)^T$.
- $P$ is called a stochastic matrix, that is $\sum_i p_{ij} = 1$.
- Find the eigenvalue of $P$ with largest eigenvalue $1$
- It does not change the order of $r$?!

$$Q = \begin{bmatrix} 0 & 1/3 & 0 & 0 & 1/2 & 1/2 \\ 1/3 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 1/3 & 0 & 1/2 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1/3 & 0 & 1/6 & 1/2 & 1/2 \\ 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/6 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 1/6 & 0 & 0 \\ 0 & 0 & 1/3 & 1/6 & 0 & 1/2 \\ 0 & 0 & 1/3 & 1/6 & 1/2 & 0 \end{bmatrix}$$

# Perturbation

**Theorem**

*Let $P$ be a stochastic matrix. Let $0 < \epsilon < 1$. Define*

$$A = (1-\epsilon)P + \epsilon \frac{1}{N} ee^T.$$

*Then $A > 0$ is an irreducible stochastic matrix. Thus, there exists a unique $r$ such that $Ar = r$. Moreover, let*

$$r^{(n+1)} := Ar^{(n)},$$

$$r^{(0)} > 0, |r^{(0)}|_1 = 1,$$

*then $r^{(n)} \to r$.*

# Challenges

- 60 Trillions of individual pages
- Size of stochastic matrix can be hundreds of thousands
- Fast algorithm

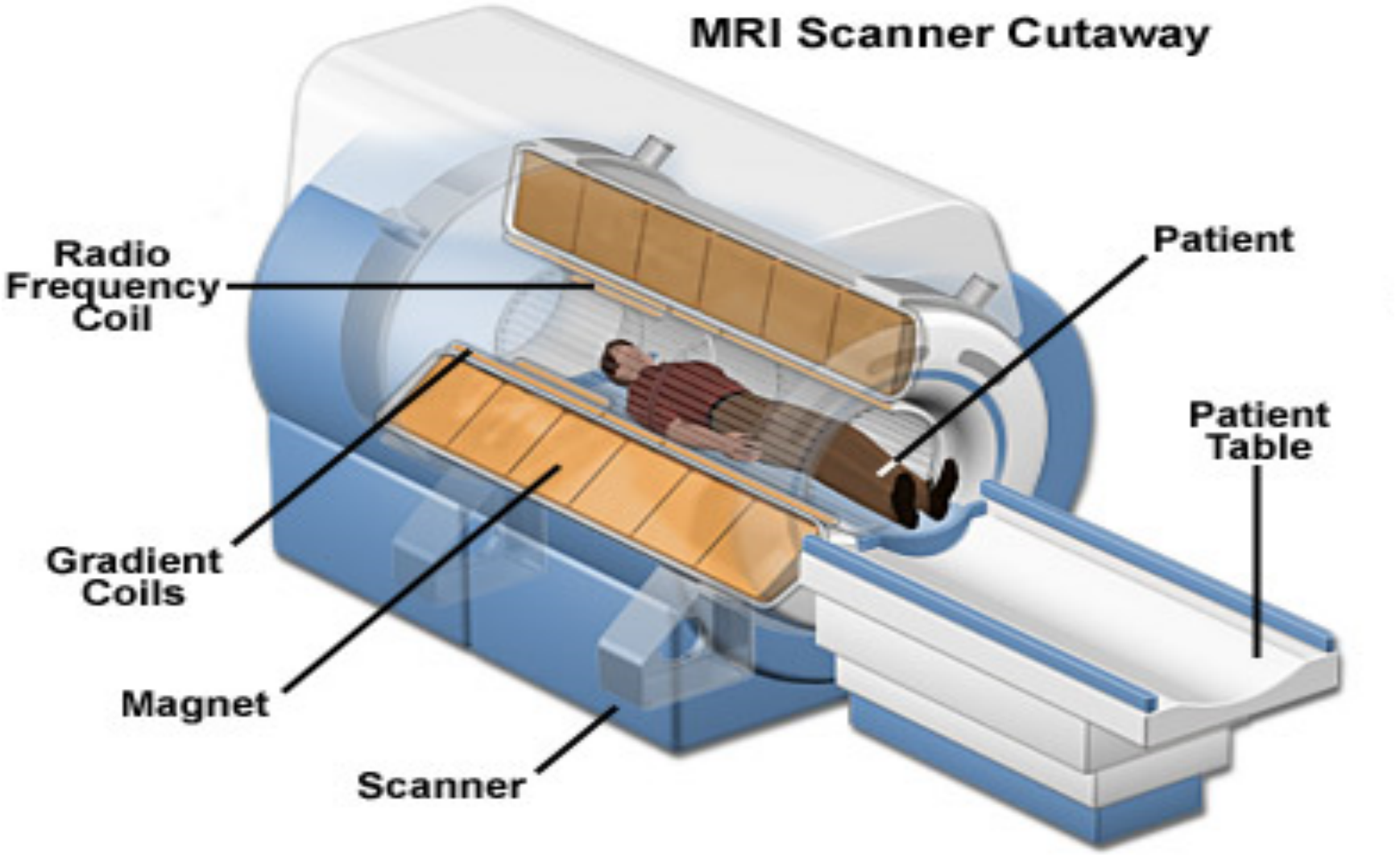
# 第四類問題

- 壓縮感知
- 影像除躁
- 影像填補

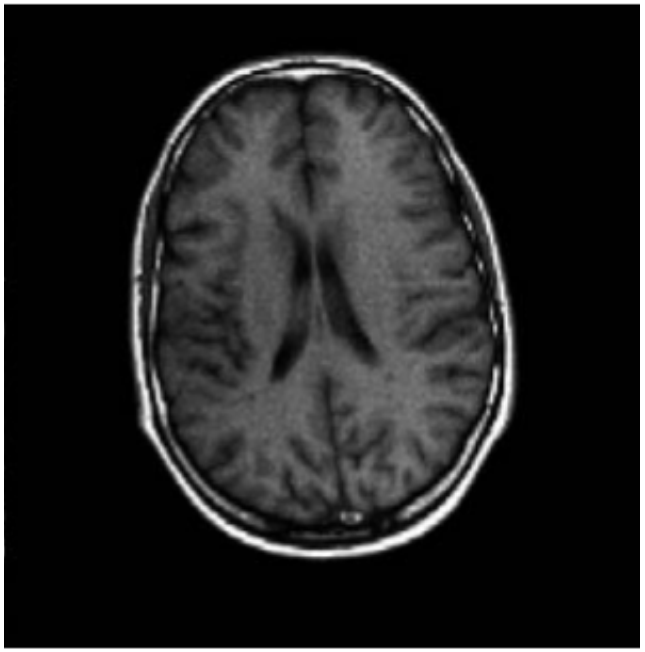
# How to reduce imaging time for MRI

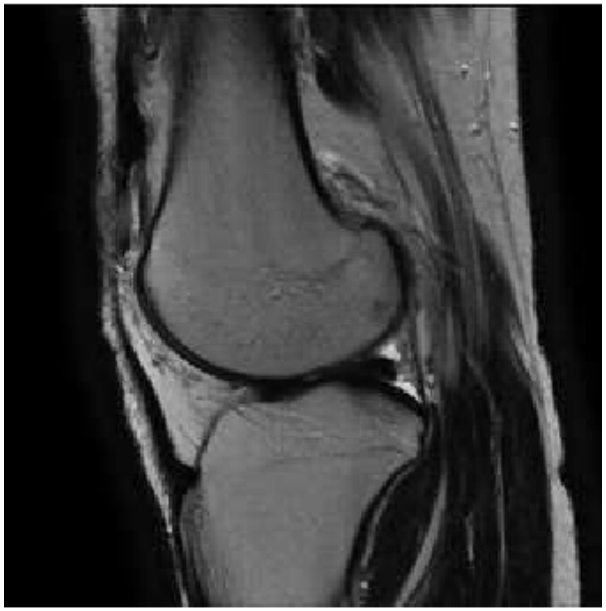
- Motivation from medical imaging
  - Low dose
  - Fast imaging
  - High resolution
- A breakthrough: Compressed Sensing

# Magnetic Resonance Imaging (MRI)



# MRI images





Typical resolution: 256 x 256 or 512 x 512

# MRI history

## The Nobel Prize in Physics 1952

"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"

**Felix Bloch**

◑ 1/2 of the prize

USA

Stanford University
Stanford, CA, USA

**Edward Mills Purcell**

◑ 1/2 of the prize

USA

Harvard University
Cambridge, MA, USA

## The Nobel Prize in Physiology or Medicine 2003

"for their discoveries concerning magnetic resonance imaging"

**Paul C. Lauterbur**

◑ 1/2 of the prize

USA

University of Illinois
Urbana, IL, USA

**Sir Peter Mansfield**

◑ 1/2 of the prize

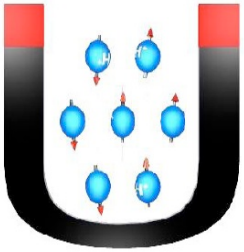
United Kingdom

University of Nottingham,
School of Physics and Astronomy
Nottingham, United Kingdom

# Basic Principles of Nuclear Magnetic Resonance

- Atoms with odd number of protons and/or neutrons poss nuclear spin angular momentum S
- Associated with S is a magnetic dipole moment

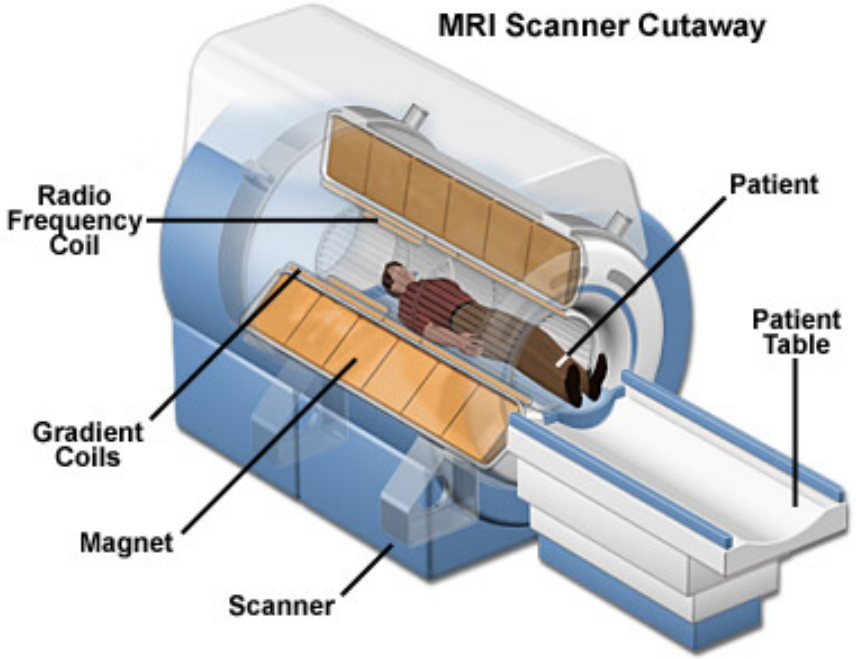

- Magnetic dipole moment rotates under external magnetic field, exhibit magnetic resonance phenomena
- The variation of rotation of spins generate magnetic fluxes and can be recorded

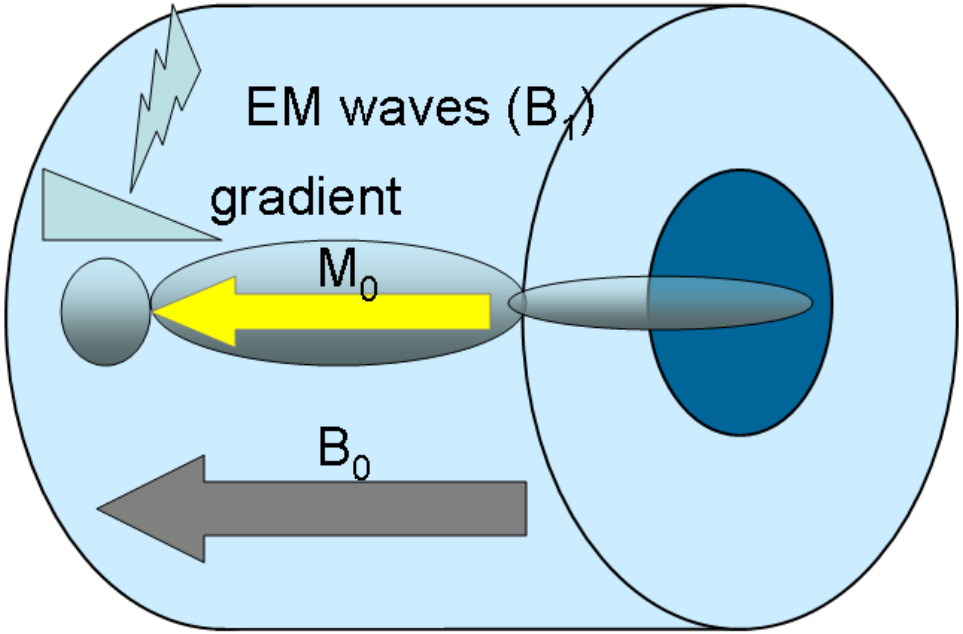


- Hydrogen H+ atoms are abundant in biological specime

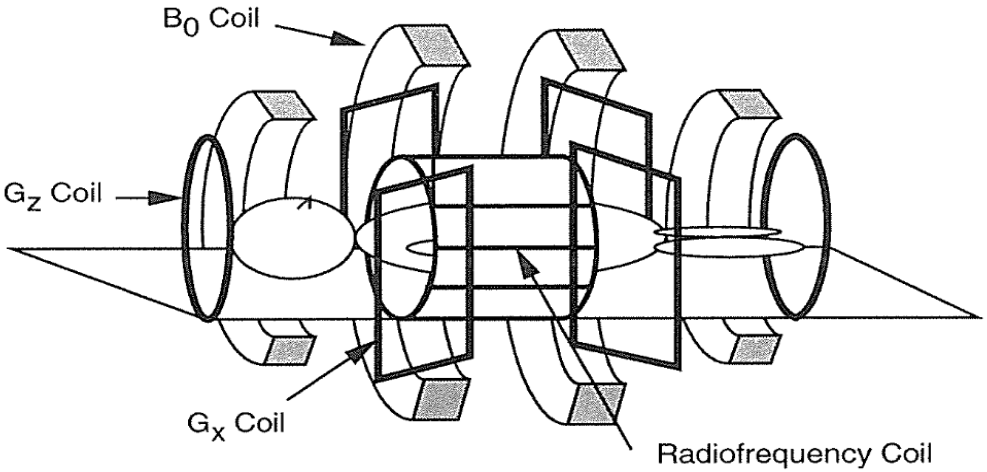
# MRI:
# use magnetic fields to perform

- Relaxation: Main field B0
- Excitation: Radio Frequency (RF) field B1
- Fourier transform: Gradient field G

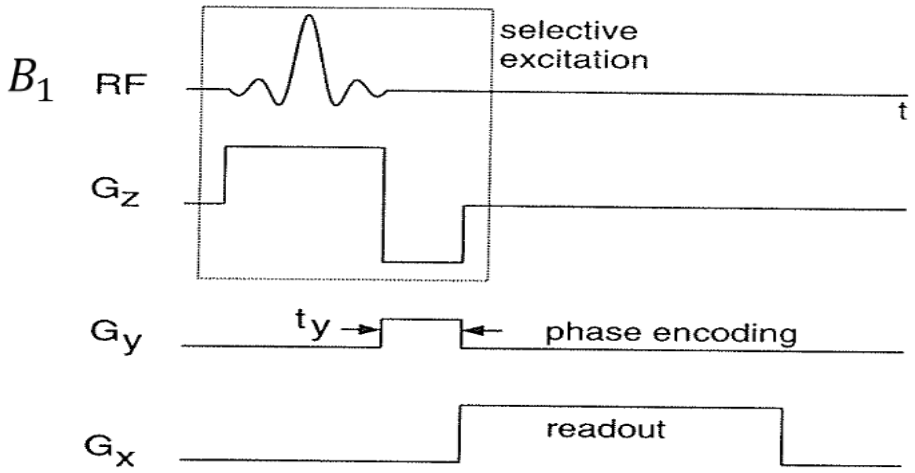


# Magnetic fields in MRI



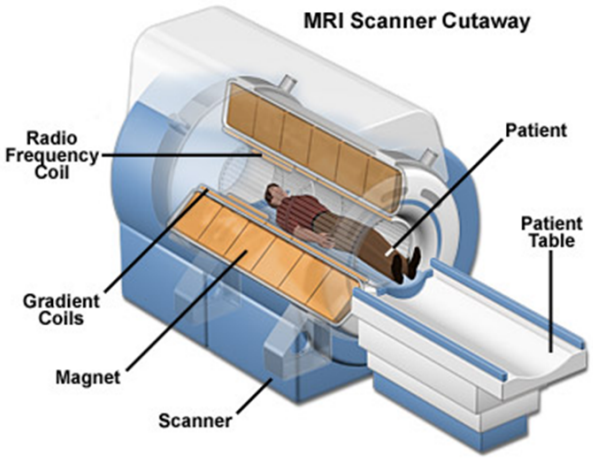

$$B(t) = B_0 + B_1(t) + G(t)$$

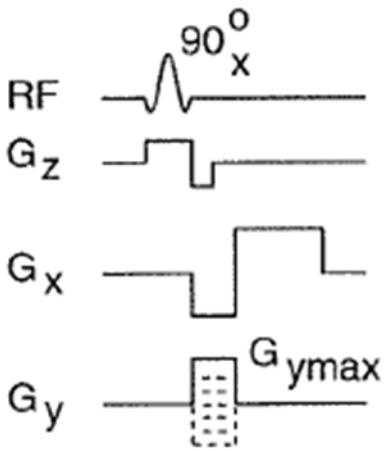


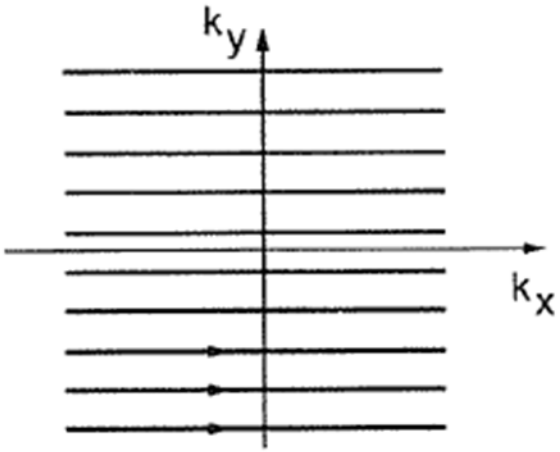
# Principle of MRI

- Relaxation: Main field B0
- Excitation: Radio Frequency (RF) field B1
- Fourier transform: Gradient field G

MRI is a Fourier integrator



# Compressive sensing

- In MRI, we want to reconstruct an $N \times N$ image $f$ from $\hat{f}(\omega)$ defined by

$$\hat{f}(\omega) = \sum_{t \in \mathbb{Z}_N^2} f(t) e^{-2\pi i \omega \cdot t/N}$$

  with $\omega \in \Omega$ with $\#\Omega << N^2$.

- Suppose $N = 512$. We choose $\Omega$ consisting 22 radial lines with 512 uniform sample points on each line.
- The reconstruction can be exact if $f$ is sparse.

E. Candes, J. Romberg, T. Tao 2006
David Donoho 2006

# Compressive sensing

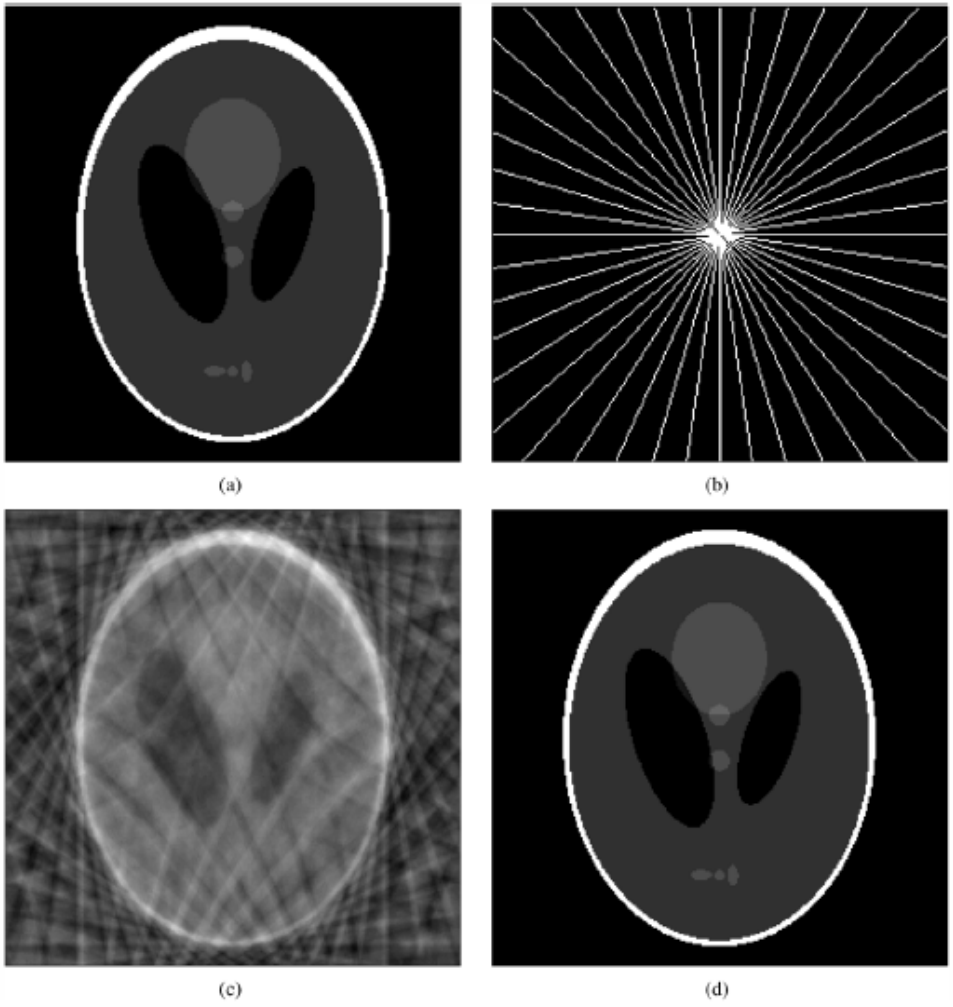



Fig. 1. Example of a simple recovery problem. (a) The Logan–Shepp phantom test image. (b) Sampling domain $\Omega$ in the frequency plane; Fourier coefficients are sampled along 22 approximately radial lines. (c) Minimum energy reconstruction obtained by setting unobserved Fourier coefficients to zero. (d) Reconstruction obtained by minimizing the total variation, as in (1.1). The reconstruction is an exact replica of the image in (a).

Candes,Romberg,Tao 2006

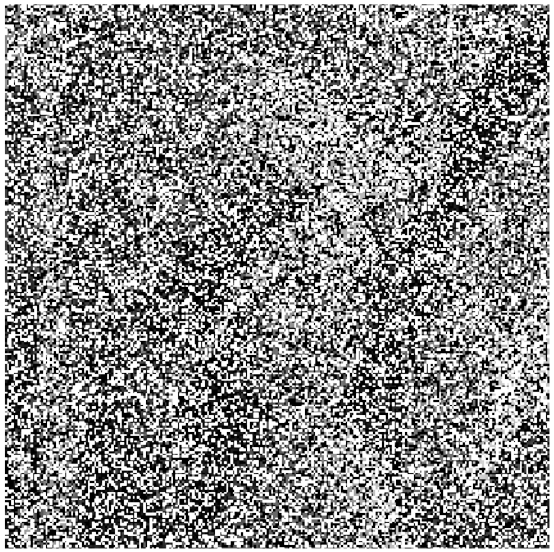
# Compressive sensing in Fourier space

- The result of filtered backprojection is poor if $\Omega$ is sparse.
- The result of TV regularization is exact if $f$ is sparse:

$$\min_u \|\nabla u\|_1 \text{ subject to } \hat{u}(\omega) = \hat{f}(\omega), \omega \in \Omega.$$

$$\|\nabla u\|_1 := \sum_{ij} |u_{i+1,j} - u_{ij}| + |u_{i,j+1} - u_{ij}|$$

# 影像除躁
# Denoising



Chan, Ho, Nikolova L1

**70% Salt-and-Pepper Noise**

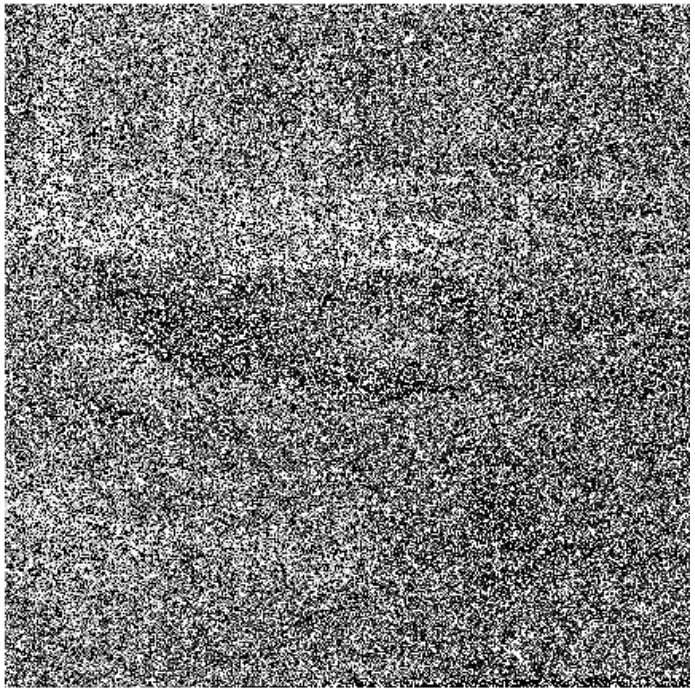
- Image model: $z = u + n$
- True image $u$, observed image $z$, noise $n$ (impulse)
- Data fitting functional

$$\|z - u\|_1 := \sum_{ij} |z_{ij} - u_{ij}|$$

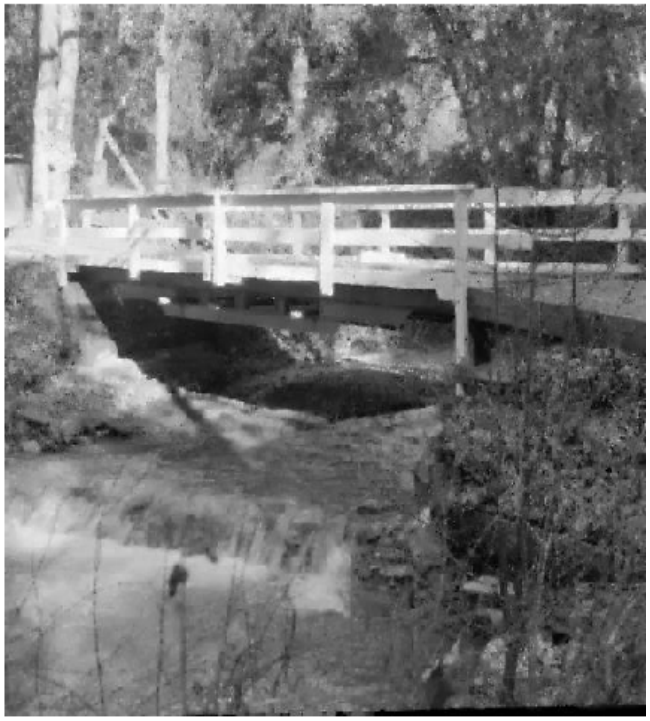
- Prior functional $\|\nabla u\|_1$.

$$\min_u \|\nabla u\|_1 \text{ subject to } \|u - z\|_1 \leq \eta.$$

# Denoising

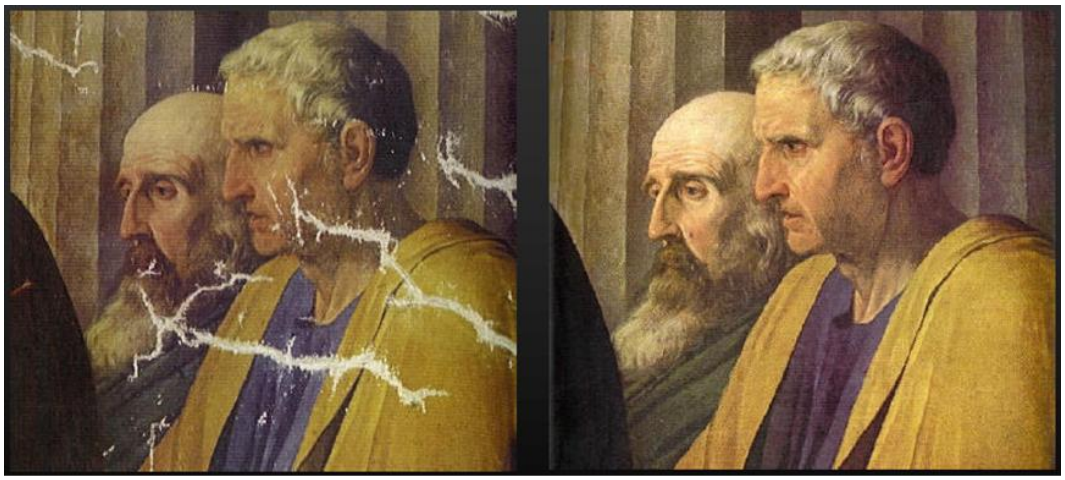


**70% Salt-and-Pepper Noise**



Chan, Ho, Nikolova

# 影像填補(Image Inpainting)



***"Image Inpainting : An Overview",***
Guillermo Sapiro


***"Fast Digital Image Inpainting",***
Manuel M. Oliveira,
Brian Bowen,
Richard McKenna
and Yu-Sung Chang

Chiu-Yen Kao

# Image Inpainting

- $z$: observed image, $D$ missing inpainting domain
- Energy functional:

$$E(u, D) = \gamma E_p + \int_{\Omega \setminus D} |u - z|^2$$

- Prior functional:

TV: $E_p = \int_\Omega |\nabla u|$

Elastica: $E_p = \int_\Omega \phi(\kappa) |\nabla u|$

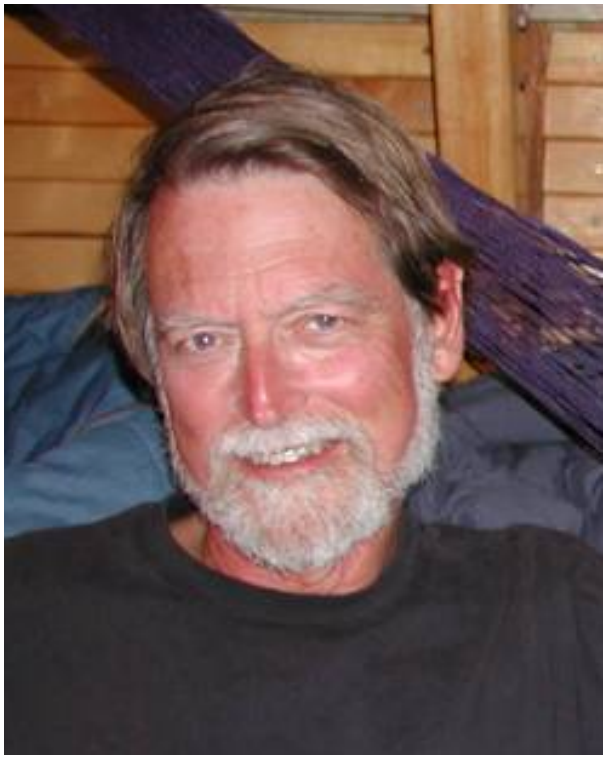
where

$$\phi(s) = \alpha + \beta s^2, \ \kappa = \nabla \cdot \left[ \frac{\nabla u}{|\nabla u|} \right].$$

# Image Processing

Stanley Osher

Tony F Chan



David Mumford

Some Applied Mathematicians in Image Processing

# Compressive Sensing (2004)

David Donoho

Emmanual Candes

Terence Chi-Shen Tao

Candes, Romberg, Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 52, NO. 2, FEBRUARY 2006 cited 4894

DL Donoho, Compressed sensing, Information Theory, IEEE Transactions on, 2006 , cited 6716

# 數學有何用

- 數學是理性思維的工具
  - 探索未知
  - 解決現實問題
  - 開創未來世界

需要數學

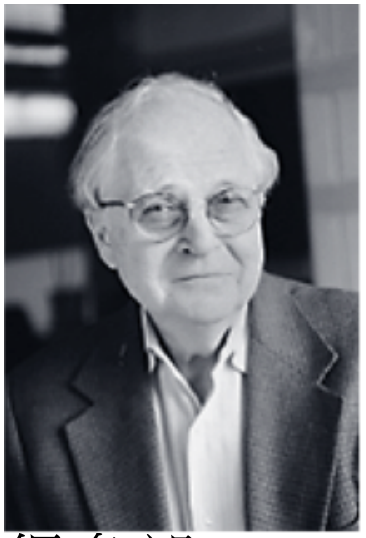
# 未來趨勢 未來挑戰

- 全球暖化
- 自動化的時代
- 都市化
- 資訊的世界
- …

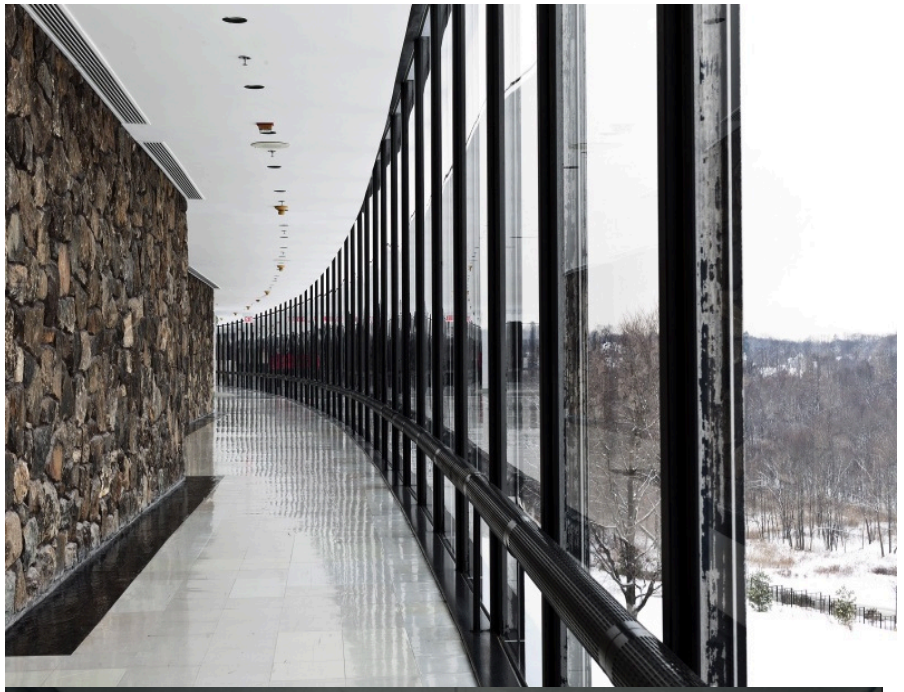
- 能源危機
- 高失業率
- 貧富懸殊

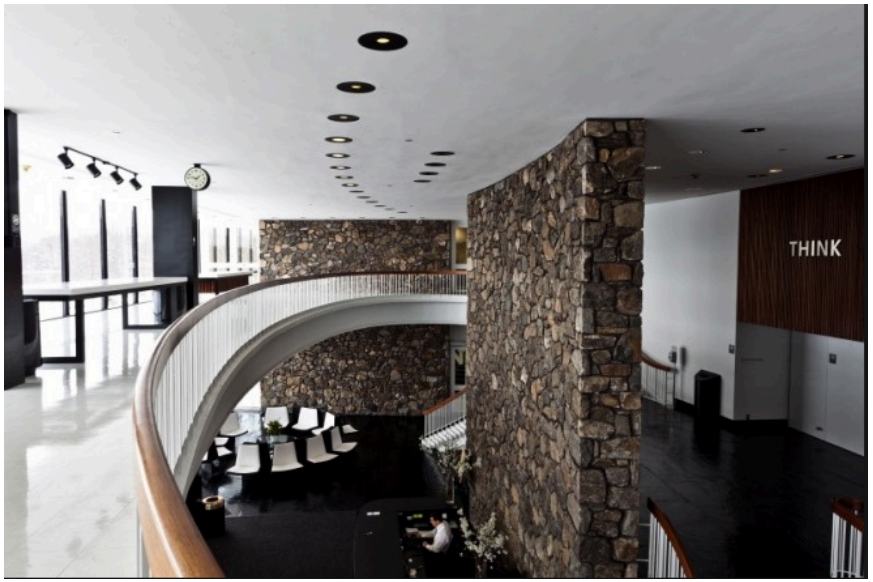
因應未來挑戰 vs 開創未來世界

# 我的一個經驗



- 1979年我在IBM的Watson Research Center 做 summer student
- 我的mentor: Williard Miranker (1956 NYU Ph.D)
- 我兩周內把他給我的問題做完了, 之後他帶我參觀很多部門
  - 在空間設計上,部門之間沒有隔閡,全部都連通,吃飯、喝下午茶也都在一起
  - Miranker因此和很多部門的人都熟識,知道很多部門的前緣研究, 包括語音部門,programming language, 還有一些物理實驗的部門 等
  - 其中最特別的是未來學的部門: 他們常與科學家與工程師互動,探 討未來科技或社會的走向,或者是想像一種新的發明,並激盪出 可行的方案




THINK

# 我的一個經驗

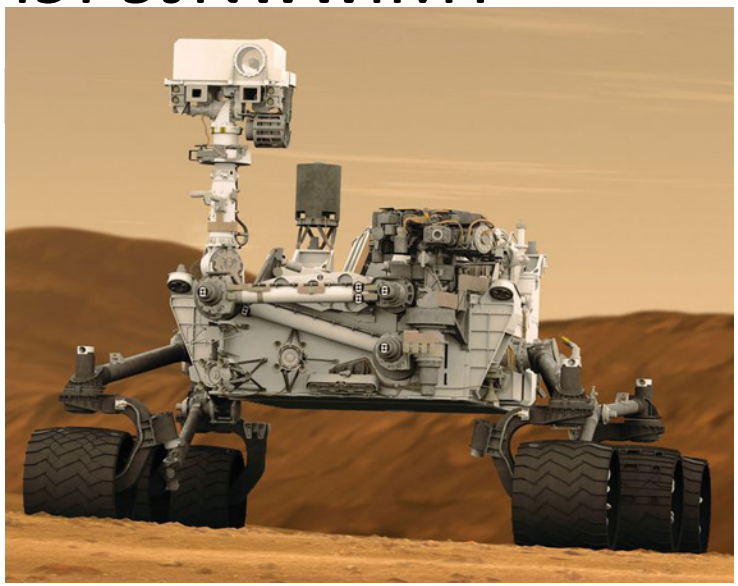
- 過去30年許多知名的科技發展都出自此,例如DES(Data EncrypOon Standard)加密演算、經典的電腦程式語言: FORTRAN(Formula TranslaOon System)、本 華‥曼德博(Benoît B. Mandelbrot)的報告書中發表了碎形 (Fractal)、磁性碟片 儲存(硬碟)、用單一個電晶體即 可記憶一個Bit的動態RAM(Dynamic Random Access Memory,DRAM)、精簡指令集電腦(RISC)架構、以及 關聯式資料庫等。IBM研究院在物理科學上也有所貢獻, 包括掃描隧道顯微鏡(簡稱:STM)以及高溫超導等,此 兩項成就都獲得了諾貝爾獎。(Wiki)

# 創造未來的一個設計

- 這是創造未來的一個設計。
- 進行前緣研究的專家們在開放的環境下, 彼此自然認識
- 有機會激盪、蘊釀、成熟思想。看到跨領域研究自然產生。

# 第二個例子: 誘發動機

- 2012年加州理工學院(Caltech) 的新生訓練
- Adam Steltzner演講: 他是噴射引擎實驗室(JPL)的登陸火星探測車(好奇號，Curiosity Rover)的計畫主持人http://www.youtube.com/watc?v=l9P9JNwwiMY
- ars

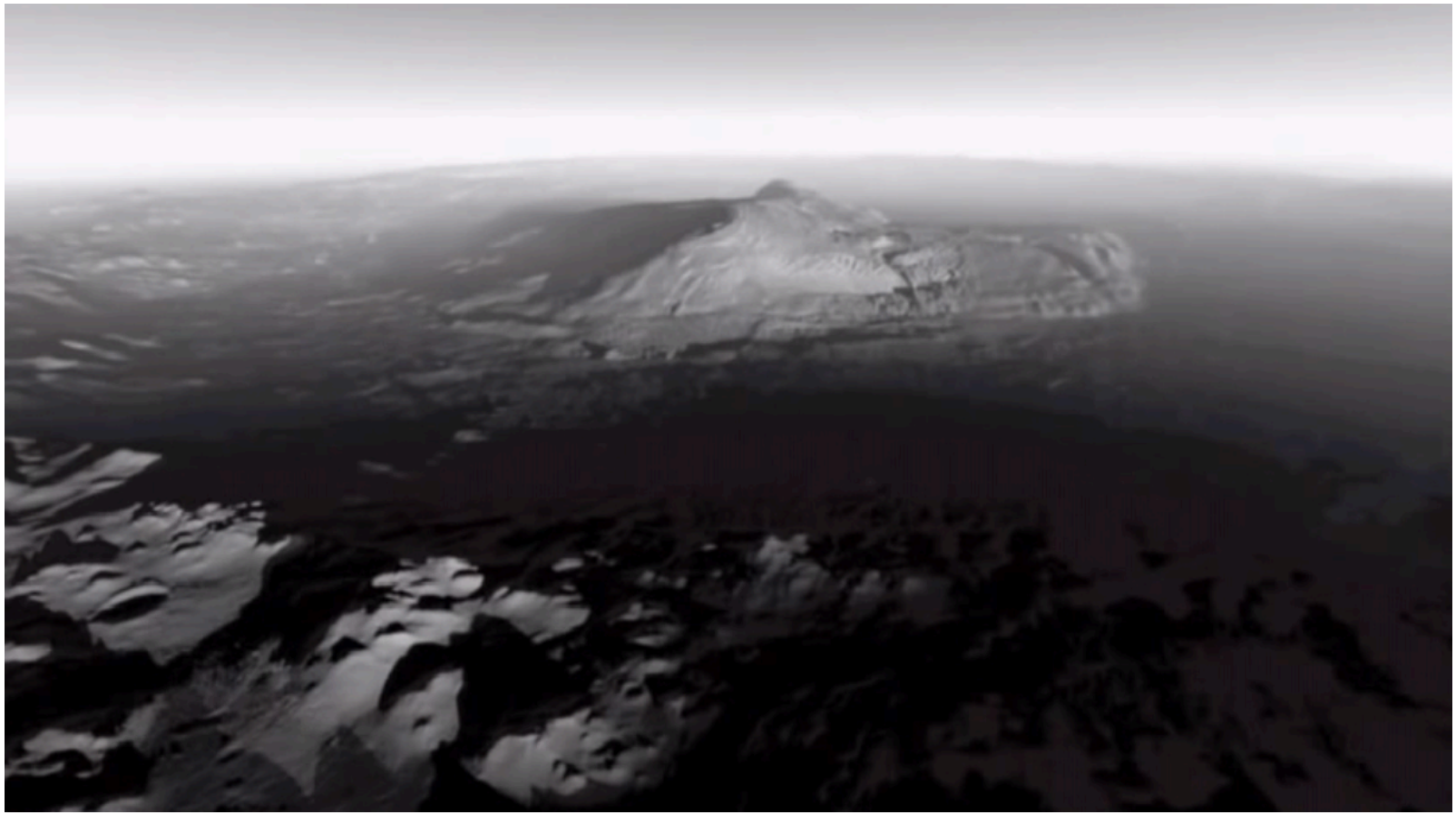


- 他小時候對星空的好奇，對人類登陸月球的興奮。
- 對探索火星的奧祕感到無比的興奮
- 紅色的火星上有沒有水呢？有無生命呢？

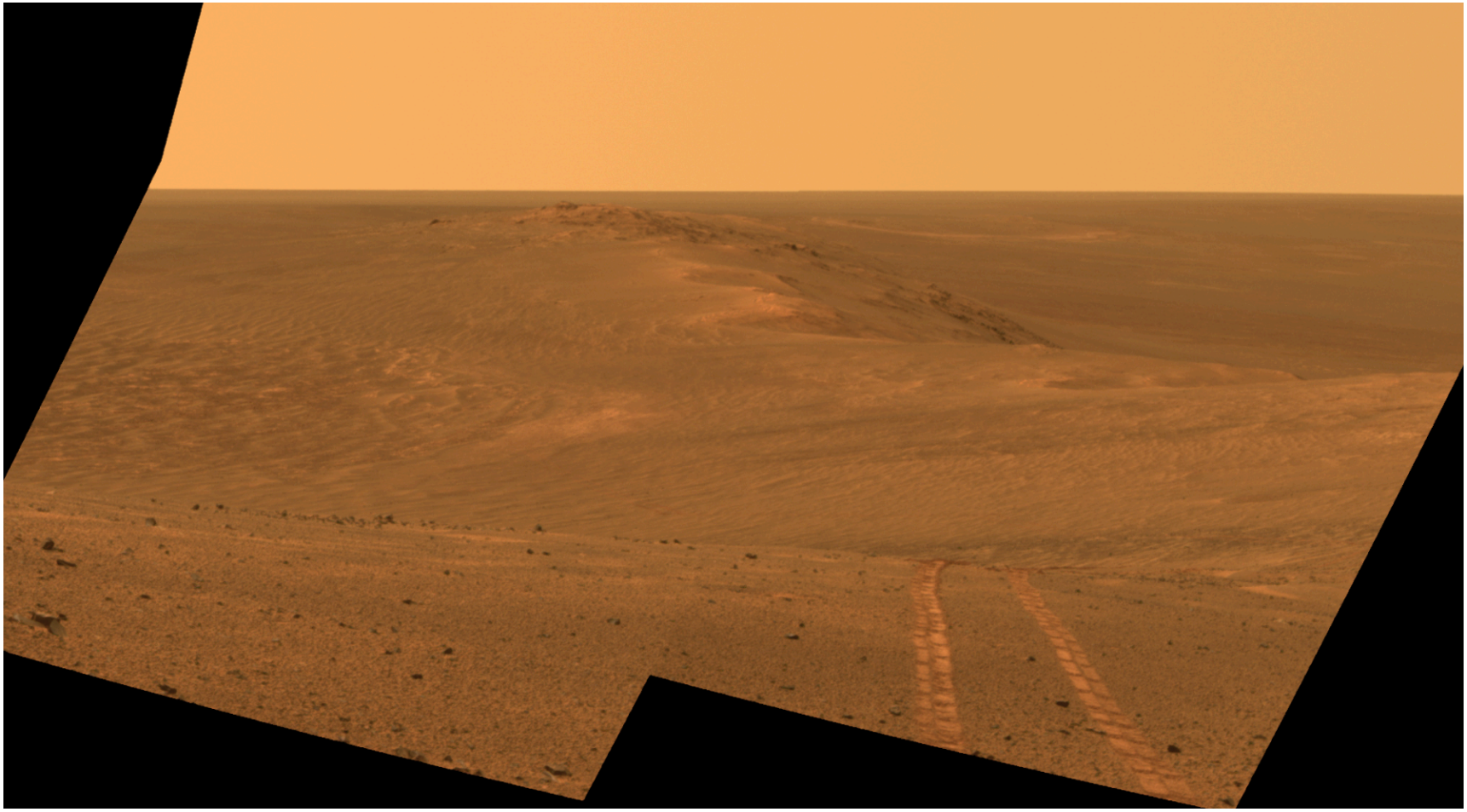
- 這是一個追求真理的好奇之旅,而這個探測車有一 輛小汽車那麼大,要零失誤地在7分鐘內自大氣層 外登陸火星地表,這是一項非常不容易的任務, 他說明許多可能的困難及他們團隊的解決之策。

# 好奇與探索

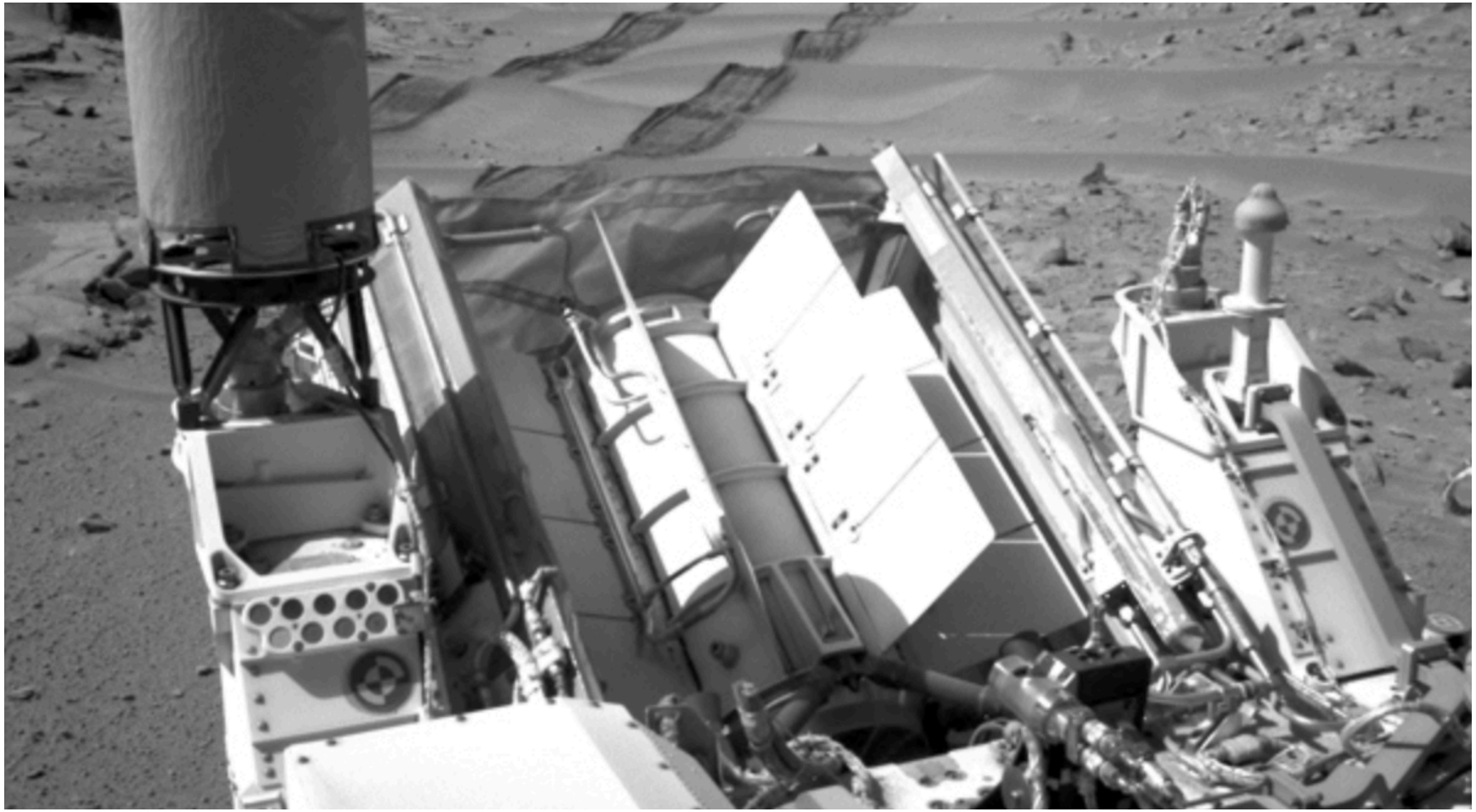
- Exploring the universe, following our curiosity.

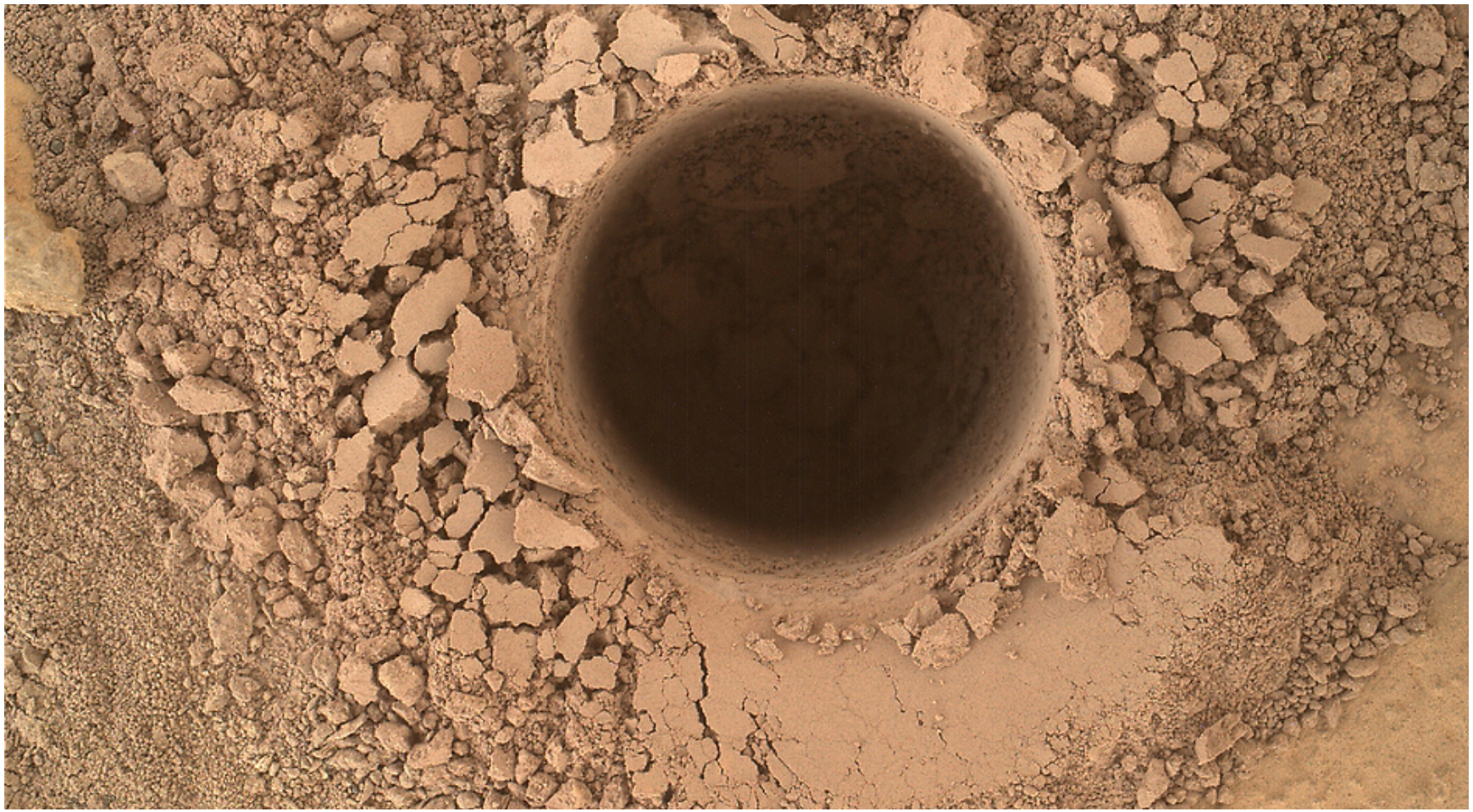


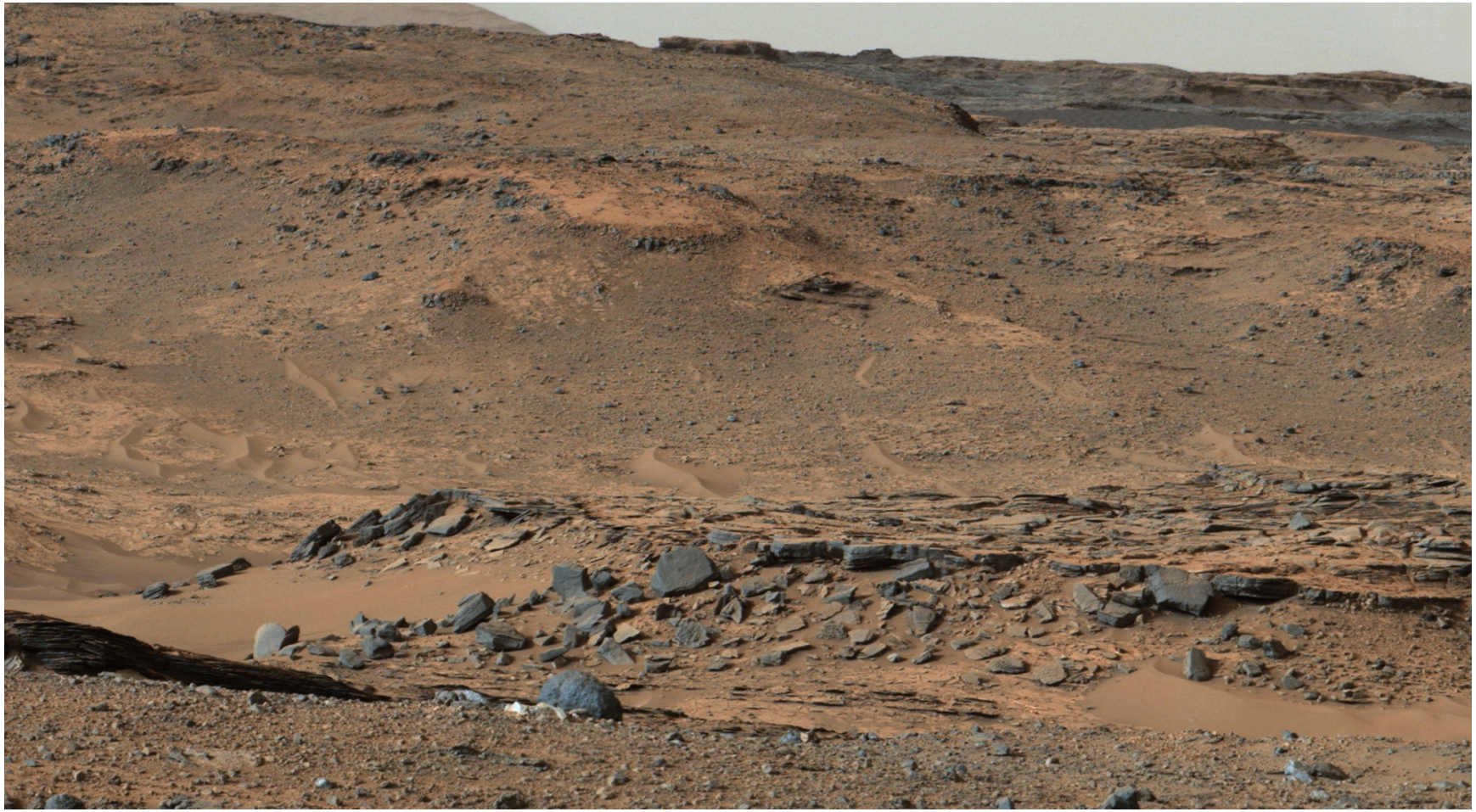
- 而就在他演講的一刻,同步放映了探測車自火星地表傳來的影像,映在三層樓高的大銀幕 上,學生仿如身歷其境,既震撼又感動。這是一 場主題為好奇、夢想、挑戰未來的大型表演,無疑的它是一場振奮新生的成功表演。



This scene from the panoramic camera (Pancam) on NASA's Mars Exploration Rover Opportunity looks back toward part of the west rim of Endeavour Crater that the Rover drove along, heading southward, during the summer of 2014.







# 好奇與探索

- 你對什麼好奇呢?
- 你做過什麼探索呢?
- 你的探索有用到數學嗎?
- 你做過數學相關的探索嗎?

# 未來與過去: 一個思維模式的差異

- 西方的電影: 有很多Science Fictions, 災難片, 很　多是以未來為背景
- 大陸的電影: 宮廷片,背景是過去,現代片
- 港片:古裝,現代片
- 台片:現代片

# 東西文化面對過去與未來的差異

- 西方文化對未知事務的好奇,對未來的夢想,對求新求變的渴望,對奇發異想的包容甚至賞識, 對科技發展的高視野,這些都是推動創新的文化元素

- 東方文化較為現世,同時也有太多過去的包袱。 在思維上較少向未來看,或幻想未來的世界。

# 開創未來

- 轉個念,多向未來看看吧。
- 年輕人,勇敢地編織夢吧。

# 高中生數學建模競賽

- Cornell University has a couple of sites for high school students
- http://www.math.cornell.edu/~mec/
- http://www.math.cornell.edu/~numb3rs/

- COMAP has High school Math Contest for Modeling
- http://www.comap.com/highschool/contests/h

- SIAM M3 challenge
- http://m3challenge.siam.org/

- WPI CIMS Industrial Mathematics Project for High School Students
- https://www.wpi.edu/academics/math/CIMS/IMPHSS/projects.html

# 大學生數學建模競賽

- COMAP MCM/ICM
- http://www.comap.com/undergraduate/contests/

- UCLA RIPS
- http://www.ipam.ucla.edu/programs/student-research-programs/research-in-industrial-projects-students-rips-2014/?tab=overview

- WPI CIMS REU
- https://www.wpi.edu/academics/math/CIMS/REU/

- NCSU REU
- https://www.math.ncsu.edu/REU/

# Moody's Mega Math Challenge®

A CONTEST FOR HIGH SCHOOL STUDENTS



**Jump to:**

- Who Can Participate?
- Dates & Forms
- Winning Solutions
- Prizes & Recognition
- Teacher-Coach Guidelines
- Modeling Handbook
- Judging
- Submit Problem Ideas




Registration will open and up-to-date information for the 2015 Challenge will post here in November. Stay tuned!

## News & Announcements...

### Challenge dates set for 2015!

**AUGUST 27, 2014**

Dates for Moody's Mega Math Challenge 2015 have been set. Challenge weekend will take place February 28-March 1, 2015. The final round of judging, including top six team presentations and awards ceremony, will be held Monday, April 27, 2015.

Registration opens in November. Stay tuned for a new look to our website and more 2015 information to be posted in October.

*read more*

## Video Gallery

**About Moody's Mega Math Challenge 2014**


## Photo Gallery

# Lunch Crunch: Can nutritious be affordable and delicious?

- First Lady Michelle Obama spearheaded an initiative on good nutrition that led to passage of the Healthy, Hunger-Free Kids Act of 2010. Implementation of the act, however, revealed the competing preferences of the school lunch program's three major stakeholders. **Students** care most about taste and quantity; **school districts** are concerned about affordability; and the **federal government,** which provides financial support, wants to promote lifelong healthy eating habits.
- Schools have seen the cost of offering lunch go up (since healthier foods are often more expensive), while participation goes down (students are less satisfied with school lunch, either because it doesn't taste as good or
  it isn't filling enough), causing a fiscal crisis for some school districts1.
- The USDA has asked your consulting firm to provide a report with mathematically founded insights into the problem; you should address at minimum the following three concerns.

- 1. **You are what you eat?** Students' caloric needs at lunch depend on how active they are, whether they eat breakfast, and a host of other factors. Develop a mathematical model that takes as input a student's individual attributes, and outputs the number of calories that a student with those attributes should eat at lunch.
- 2. **One size doesn't necessarily fit all.** The guidelines dictated by the Healthy, Hunger-Free Kids Act of 2010 are based on meeting the needs of an "average student"2. However, meeting the average need may not necessarily be the right amount for many students. Now that you've identified attributes that affect caloric needs at lunch, create a model to determine the distribution of U.S. high school students among each of these categories. If every student eats the standard school lunch, what percentage of students will have their caloric needs met at lunch?
- 3. **There's no such thing as a free lunch.** A sample school district has a weekly budget of $6 per student
  for the purchase of food only. Leverage math modeling to develop a lunch plan (using food categories) that stays within the budget, meets the nutritional standards and appeals to students. What changes would you make if your budget was increased by $1?

You may want to take into account how your model could be applied across different geographic and/or socio-economic regions.Your report to the USDA should include a one-page summary of your findings.
You may find the following websites helpful:

http://www.globalrph.com/estimated_energy_requirement.htm
http://www.cdc .gov/mmwr/pdf/ss/ss5905.pdf
http://www.cdc.gov/growthcharts/charts.htm#Set3
http://www.amstat.org/censusatschool/about.cfm

Winning team

# 如何做好準備

- 數學物理兼備
- 學會寫程式 (matlab)
- 多接觸應用課題，從實作當中學習