數學與當代生活

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影響當代生活的兩個重要發明

- 電腦: 泛指具運算能力的各種裝置
- 互聯網(Internet)
- 想想看，如果你一天沒手機，沒電腦，沒網路，你會怎麼樣。

在電腦裡運作的是 算法、數學

用六個例子來闡述

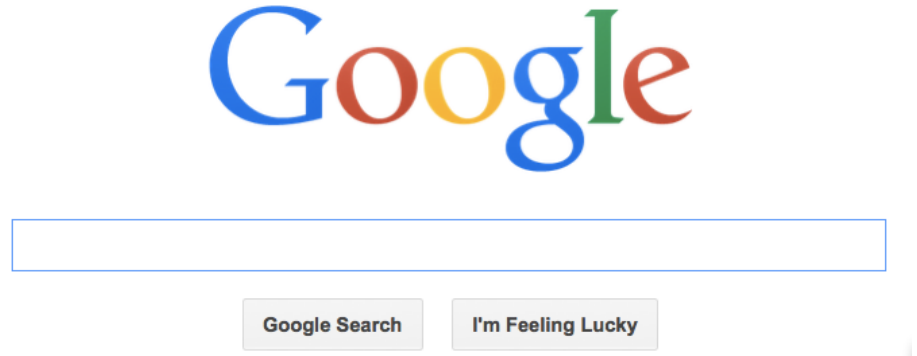
- How google works
- How Amazon recommends books
- What is digital image processing
- How X-ray CT works
- How to reduce imaging time in MRI
- How Snow is simulated in the movie FROZEN

現代生活中的數學

- How google works
- How Amazon recommends books
- What is digital image processing
- How X-ray CT works
- How to reduce imaging time in MRI
- How Snow is simulated in the movie FROZEN

Google 搜尋引擎

- 訊息搜尋
- 知識搜尋 (Wiki)
- 地圖搜尋
- 拉近了距離 (空間、時間)
- .....



Google 改變我們的生活

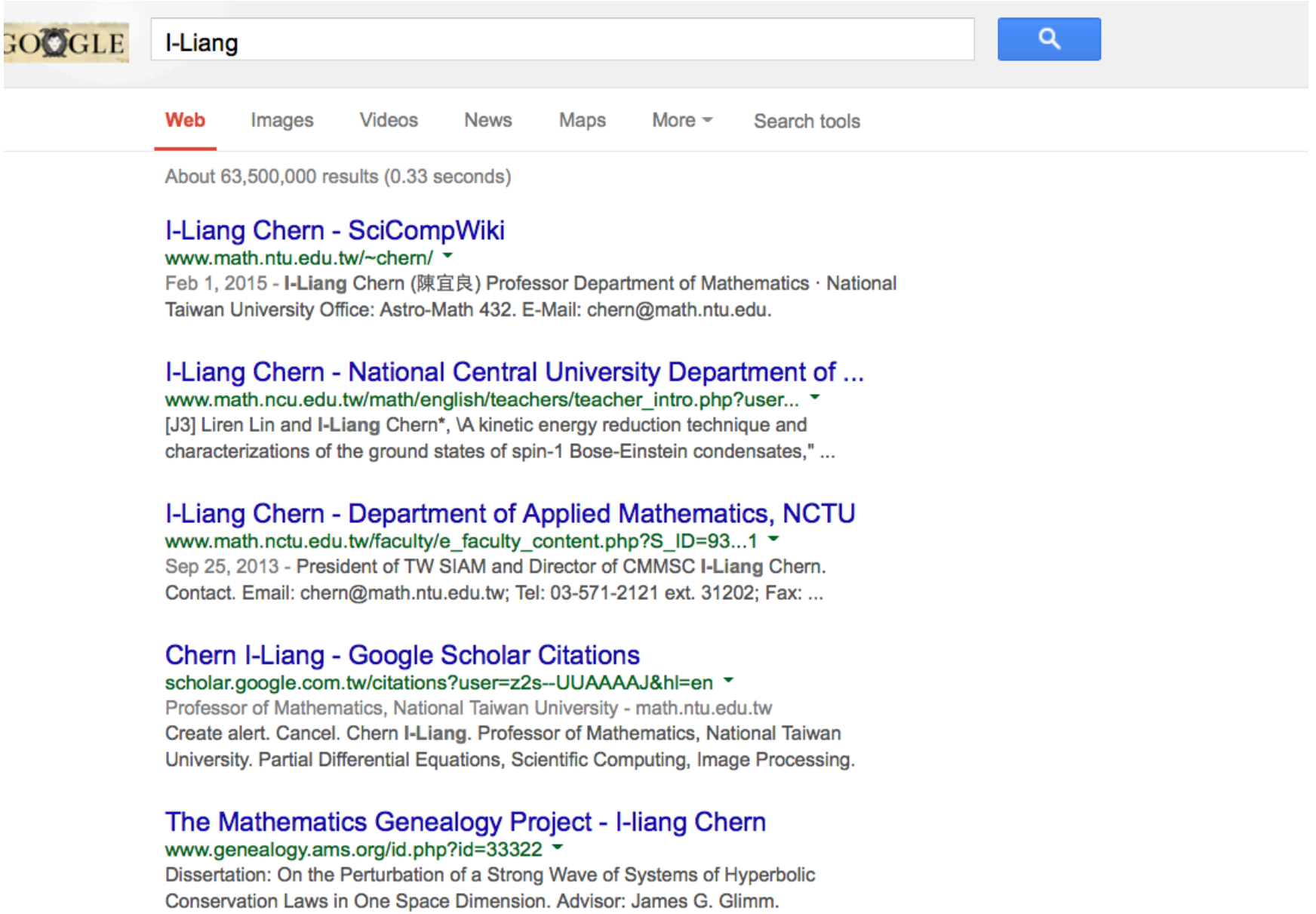
How google works

- 爬挖資料 (Crawling)
- 建立索引 (Indexing) 60 trillion pages
- 排序(Search algorithm and page ranking)

Google 的核心技術: PageRank

- 它是1996年兩名史丹福大學的研究生Sergey Brin , Larry Page 所提出的搜尋引擎
- 1998年他們成立了Google 公司。

Page and Brin in 2003


GOOGLE
I-Liang
Web
Images
Videos
News
Maps
More
Search tools
About 63,500,000 results (0.33 seconds)
I-Liang Chern - SciCompWiki
www.math.ntu.edu.tw/~chern/
Feb 1, 2015 - I-Liang Chern (陳宜良) Professor Department of Mathematics · National Taiwan University Office: Astro-Math 432. E-Mail: chern@math.ntu.edu.
I-Liang Chern - National Central University Department of ...
www.math.ncu.edu.tw/math/english/teachers/teacher_intro.php?user...
[J3] Liren Lin and I-Liang Chern*, \A kinetic energy reduction technique and characterizations of the ground states of spin-1 Bose-Einstein condensates," ...
I-Liang Chern - Department of Applied Mathematics, NCTU
www.math.nctu.edu.tw/faculty/e_faculty_content.php?S_ID=93...1
Sep 25, 2013 - President of TW SIAM and Director of CMMSC I-Liang Chern. Contact. Email: chern@math.ntu.edu.tw; Tel: 03-571-2121 ext. 31202; Fax: ...
Chern I-Liang - Google Scholar Citations
scholar.google.com.tw/citations?user=z2s--UUAAAAJ&hl=en
Professor of Mathematics, National Taiwan University - math.ntu.edu.tw
Create alert. Cancel. Chern I-Liang. Professor of Mathematics, National Taiwan University. Partial Differential Equations, Scientific Computing, Image Processing.
The Mathematics Genealogy Project - I-liang Chern
www.genealogy.ams.org/id.php?id=33322
Dissertation: On the Perturbation of a Strong Wave of Systems of Hyperbolic Conservation Laws in One Space Dimension. Advisor: James G. Glimm.

Introduction

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Rank	
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Field	Scientific Computing、Numerical Partial D ifferential Equations、Image processing、Partial Differential Equations
Experience	Professor National Chiao Tung University 2012~2014 Director Center of Mathematical Modeling and Scientific Computing National Chiao Tung University 2012~2014 Professor National Taiwan University 1991-2012 Chairman (Math. Dept) National Taiwan University 1999-2002 Visiting Professor Chinese University of Hong Kong Jan.-June, 2009 Visiting Professor Nat'l Univ. of Singapore summer, 1998 Visiting Professor U.C. Berkeley 1996-1997 Adjunct assistant Prof. U. of Chicago 1990-1991 Assistant Mathematician Argonne National Lab. 1989-1991 Research Assistant Prof. Courant Institute 1987-1989 Research Associate Fellow Academia Sinica 1983-1987 Post-doctoral Fellow MSRI, Berkeley Feb.-Aug. 1984

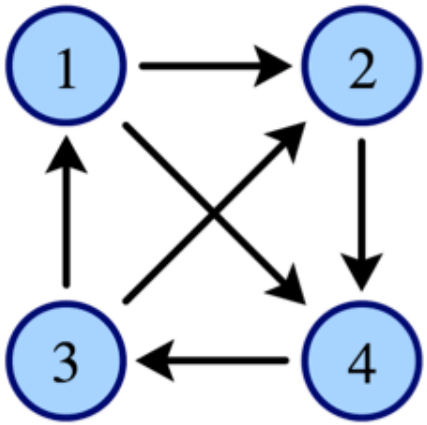
Faculty

- Sheu, shuenn Jyi
- I-Liang Chern
- Jann-Long Chern
- Chia-Chang Hsiao
- Chin-Yuan Lin
- Ming-Guang Leu
- Chin-Cheng Lin
- Cheng-Hsiung Hsu
- Suh-Yuh Yang
- Hong-Gwa Yeh
- Yen-Mei J. Chen
- Duy-Minh Nhieu
- Hwa-Long Gau
- Meng-Kai Hong
- Mei-Lin Yau
- Fang, Xiang
- I-Feng Chao
- Jui-Hsin Chen
- Jenn-Hwa Yu
- Wei-Chang Shann

PageRank: 網頁連結的分析

- Each page (with inquiry words) a node of a graph and is ordered from $1, ..., n$
- N_j: the number of pages that j is linked to (out-link).
- Define $Q_{ij} = 1/N_j$ if there is a link from j to i.

$$\sum_i Q_{ij} = 1.$$



$$Q = \begin{bmatrix} 0 & 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1 & 0 & 0 \end{bmatrix}$$

Brin和Page的貢獻

- Let $r_i \geq 0$: the importance of page i to the inquiry.
- Normalized by $\sum_j r_j = 1$.
- The importance r_i is weighted by

$$r_i = \sum_{j=1}^{N} \frac{r_j}{N_j} = \sum_{j=1}^{N} Q_{ij} r_j, \quad i = 1, ..., N$$

- It is an eigenvalue problem!

$$\boxed{Qr = r.}$$

隨機漫步的解釋

- $(r_1, \cdots, r_n)^T$ is a distribution of the importance of each page for a particular inquiry.
- Start from, say $r^{(0)} = (\frac{1}{n}, \cdots, \frac{1}{n})^T$, perform Qr^0.
- We can continue this process

$$r^{(n+1)} = Qr^{(n)},$$

  This is called a random walk.
- Hopefully $r^{(n)}$ converges to r with

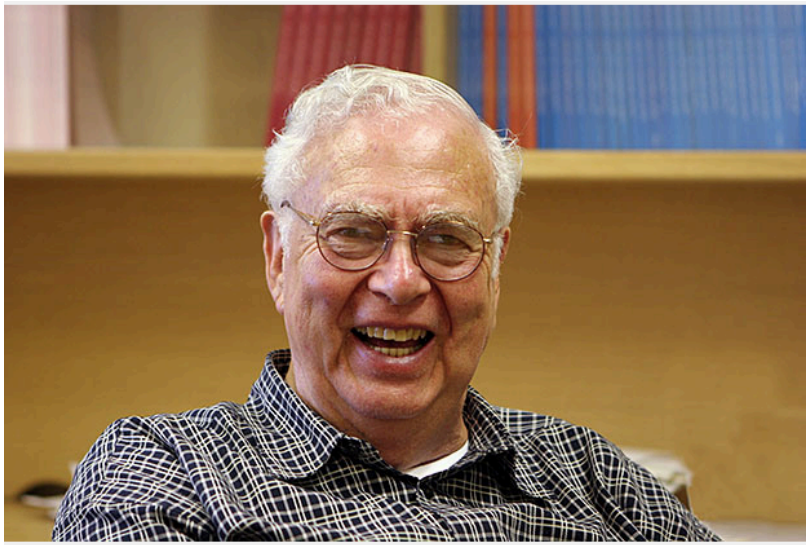
$$Qr = r.$$

  If so, then the ranking is determined by the order of magnitudes of r_i.

兩個問題

- 隨機漫步可能陷在某一網頁出不去
- 隨機漫步可能不收斂

Gene Golub, Professor at Stanford



Gene Golub, July 2005 by Nick Higham.

$$Q = \begin{bmatrix} 0 & 1/3 & 0 & 0 & 1/2 & 1/2 \\ 1/3 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 1/3 & 0 & 1/2 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1/3 & 0 & 1/6 & 1/2 & 1/2 \\ 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/6 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 1/6 & 0 & 0 \\ 0 & 0 & 1/3 & 1/6 & 0 & 1/2 \\ 0 & 0 & 1/3 & 1/6 & 1/2 & 0 \end{bmatrix}$$

收斂定理

Theorem

Let P be a stochastic matrix. Let $0 < \epsilon < 1$. Define

$$A = (1-\epsilon)P + \epsilon \frac{1}{N} ee^T.$$

Then $A > 0$ is an irreducible stochastic matrix. Thus, there exists a unique r such that $Ar = r$. Moreover, let

$$r^{(n+1)} := Ar^{(n)},$$

$$r^{(0)} > 0, |r^{(0)}|_1 = 1,$$

then $r^{(n)} \to r$.

Google 公司現況

- Google 公司在1998由Brin, Page二人成立，2001年加入Schmidt，成為鐵三角，三人於2004年協議20年內不拆夥。
- 2014年營業額600億美金，稅前利潤172億，淨利潤139億，資產1311億。九成收入來自廣告系統。
- 全球設有12個資料中心，每天處理數十億個搜尋要求。
- 公司使命是:整合全球範圍的信息，使人人皆可訪問並從中受益。
- 公司非正式口號是: Don’t be evil.
- 簡單但關鍵的數學，改變了世界。

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How Amazon recommends books

- 亞馬遜網上書店 Amazon.com 是美國最大的一家網路電子商務公司，是網路上最早開始經營電子商務的公司之一，亞馬遜書店成立於1995年，一開始只經營網路的書籍銷售業務，現在則擴及了範圍相當廣的其他產品，包括了DVD、音樂光碟、電腦、軟體、電視遊戲、電子產品、衣服、傢具等等。
- 亞馬遜會根據你購買或評比過的書籍，推薦你相關的書籍

Collaborative Filtering

- Given (incomplete) data of rating
- Determine whether Albert will like or dislike item 5

	Item1	Item2	Item3	Item4	Item5
Albert	3	4	5	3	?
User1	5	2	4	4	3
User2	3	2	2	1	5
User3	2	5	5	3	4
User4	4	1	3	4	2

Rating by correlation-weights

- Define similarity between two user a, b by the correlation

$$\text{sim}(a,b) = \frac{\sum_p (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_p (r_{a,p} - \bar{r}_a)^2}\sqrt{\sum_p (r_{b,p} - \bar{r}_b)^2}}$$

- Define prediction of rating of user a on item p by

$$\text{pred}(a,p) = \bar{r}_a + \frac{\sum_{b \in N} \text{sim}(a,b) * (r_{b,p} - \bar{r}_b)}{\sum_{b \in N} \text{sim}(a,b)}$$

	Item1	Item2	Item3	Item4	Item5
Albert	3	4	5	3	?
User1	5	2	4	4	3
User2	3	2	2	1	5
User3	2	5	5	3	4
User4	4	1	3	4	2

Sim=0.55

Sim=0.00

Sim=0.87

Sim=-0.4

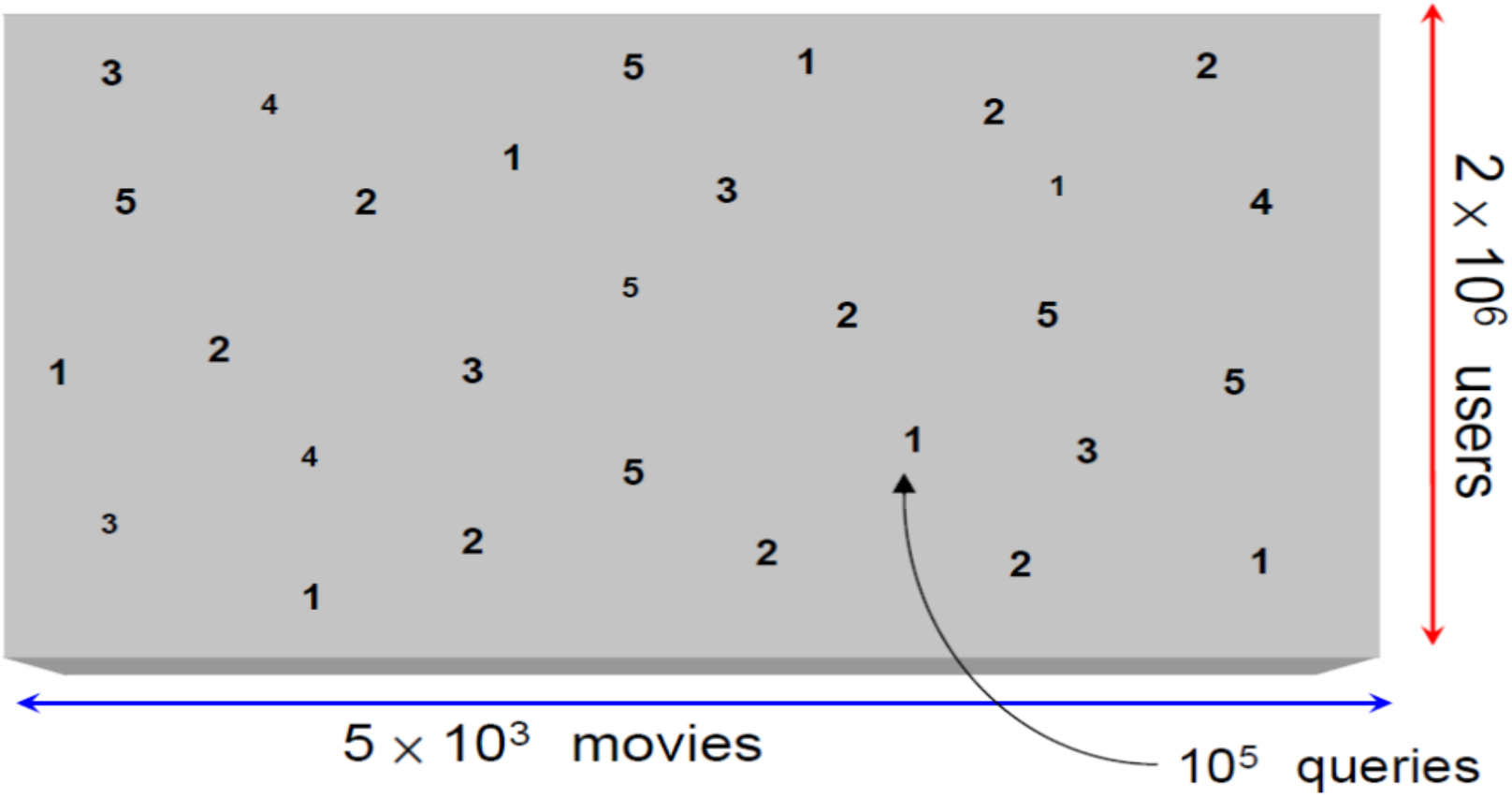
Reproduce from Powerpoint-Slides for Recommender Systems - An Introduction

Amazon 的現況

- 1994年成立，目前是全球最大的互聯網零售商之一。
- 產品:
  - 零售:圖書、軟體、音樂、家電、食品、日用品等等
  - 電子書閱讀器Kindle, 平版電腦
  - 雲端計算服務
- 規模:
  - 營業額: USD610.9億(2012)

另一作法: 矩陣完備化

- Given less than 1% of books ratings
- Goal: predict missing ratings



Netflix Problem

- Netflix是一家美國公司，提供北美地區綫上播放DVD的出租業務。
- 2011年4月，Netflix宣布在美有2360萬用戶，而全世界則超過2600萬，可提供10萬種DVD選擇。
- Netflix會根據用戶訂閱過或評比過的資料提供推薦影片 。

Netflix獎

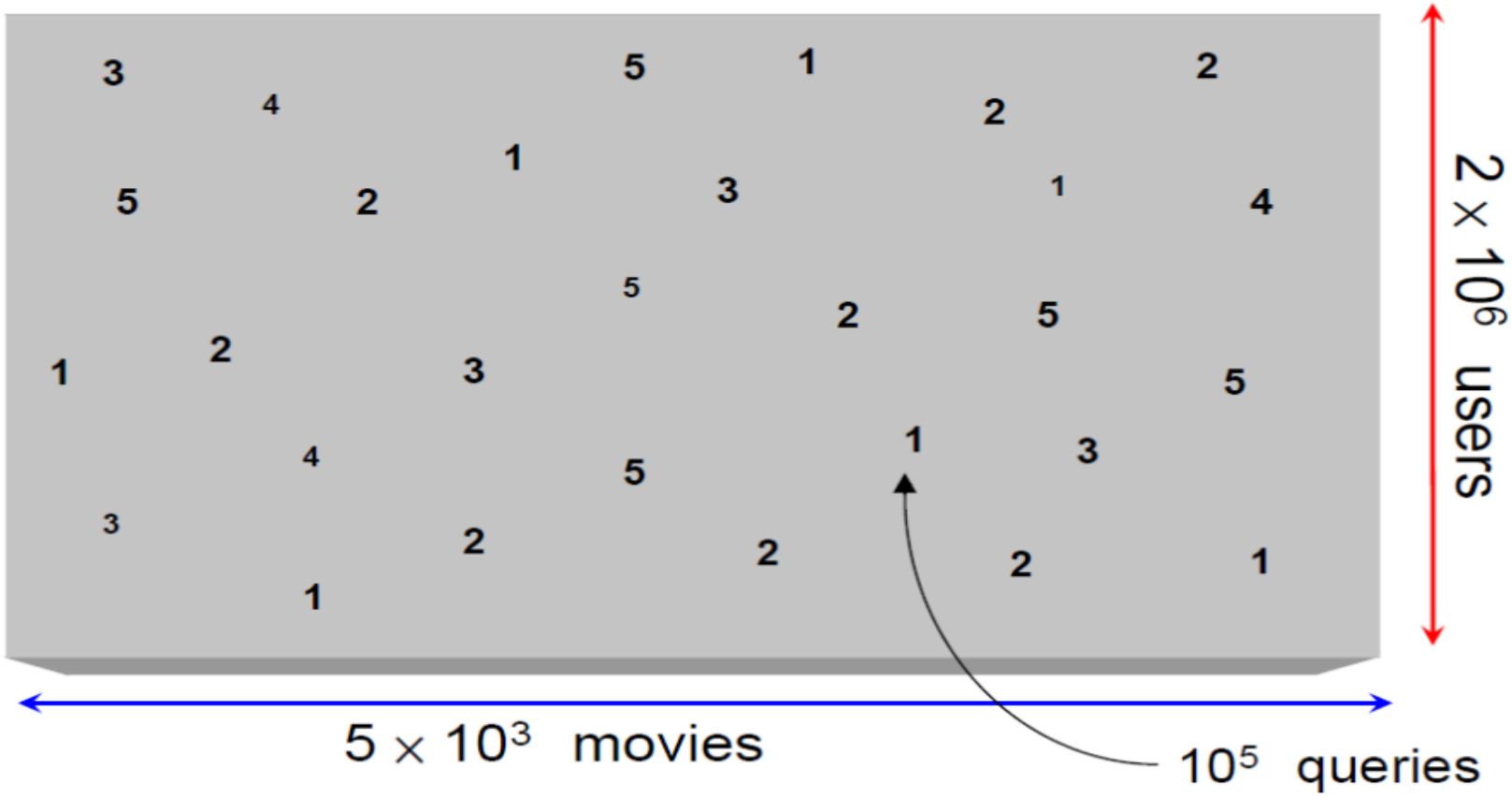
- Netflix為追求最佳collaborative filtering算法的獎，獎金一百萬美金，由Netflix公司提供。
- 比賽方式是所提供的方法要比該公司用戶的方法(Cinematch)好10%. 度量方法是RMSE (root-mean-square-error).

Netflix問題

- 資料格式: <用戶，電影，評分日期，評分>
- 訓練集: 99,072,112筆
- 小考集: 1,408,342筆
- 比賽集: 1,408,342筆
- 2009/9/18 Netflix公司宣布``BellKor‘s Pragmatic Chaos’’團隊贏得該年的Netflix獎

矩陣完備化

- Given less than 1% of movie ratings
- Goal: predict missing ratings



矩陣完備化

Hypothesis: only small number of independent subclasses.

- Model 1:

  同類相聚

  $$\min \text{rank}(X) \text{ subject to } X_{ij} = M_{ij} \text{ for } (i,j) \in E$$

  NP-hard

- Model 2:

  $$\min \|X\|_* \text{ subject to } X_{ij} = M_{ij} \text{ for } (i,j) \in E.$$

  $$\|X\|_* = \sum_i \sigma_i(X) (\text{singular values of } X).$$

  Convex optimization, easy.

矩陣完備化

- 可用來作推薦系統
- 也可用來作分類

Netflix的現況

- 2015年，全球已有6000萬個訂戶，分佈到40個國家
- 產品: 互聯網媒體播放服務，DVD光碟出租
- 規模:
  - 2014營業額: USD55億
  - 2014淨利: 2.26億

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影像處理

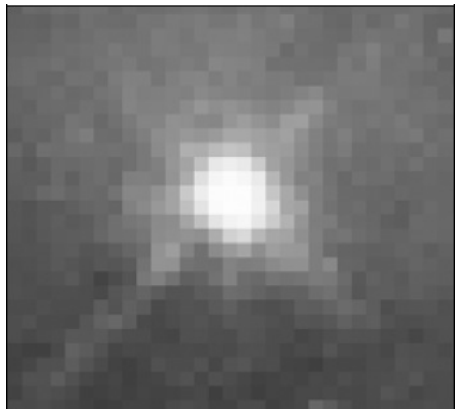
- What is digital Image?
- Image Enhancement
  - Contrast Enhancement
  - Image Denoising
  - Image Deblurring
- Image Inpainting
- Image segmentation
- Image registration
- Image representation
- Image compression

什麼是數位影像?

1. What is a digital image?

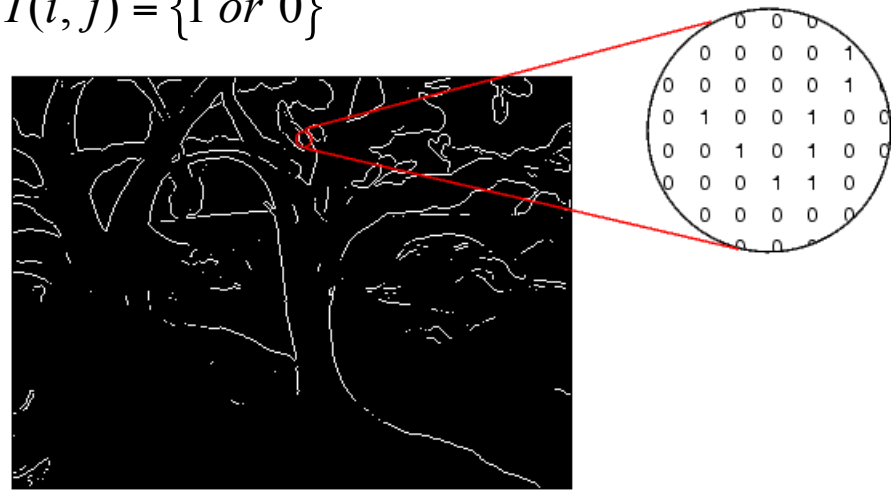
$$I:\Omega \to R \xrightarrow{\text{sampling, quantized}} I_d:\{1 \le i \le m, 1 \le j \le n\} \to R_k, 1 \le k \le l$$

A digital image Is an array, or a matrix , of square pixels (picture elements) arranged in columns and rows.



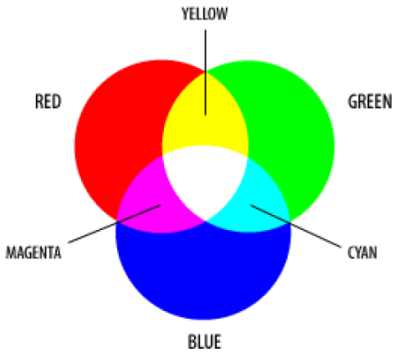
a. Binary Image (logical array)

$$I(i,j) = \{1 \; or \; 0\}$$



Chiu-Yen Kao,matlab

什麼是數位影像?



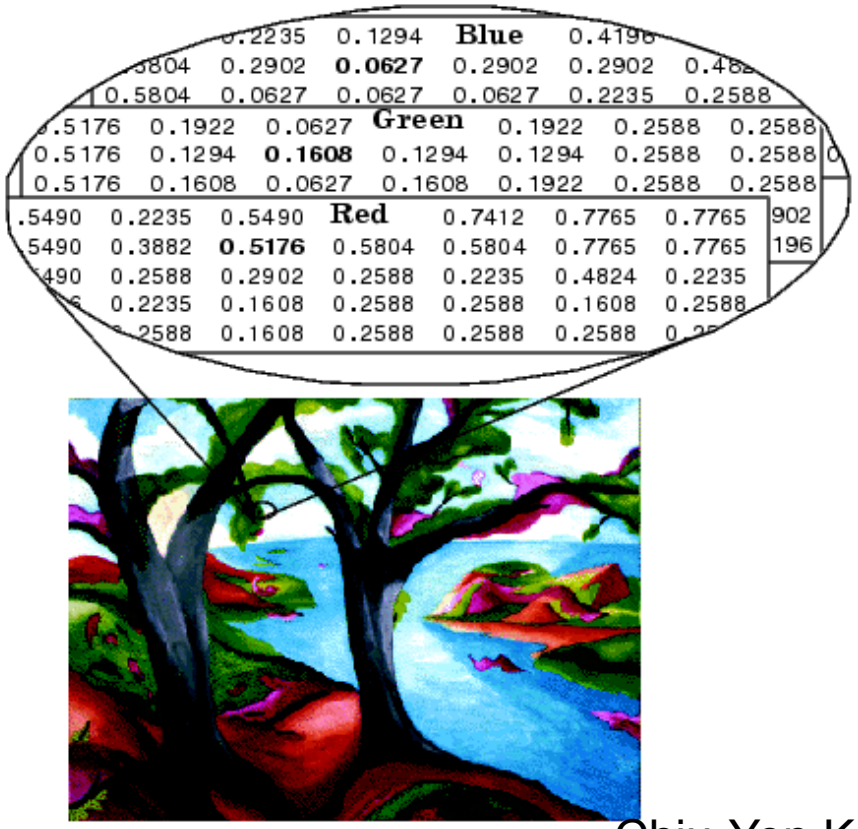
b. **Intensity Image**

8 bit (uint8, 0-255), 16 bit (uint16, 0-65535) and double ([0 1])

a. **color Image**

RGB:

24 bit = 256^3 ~ 16 million colors

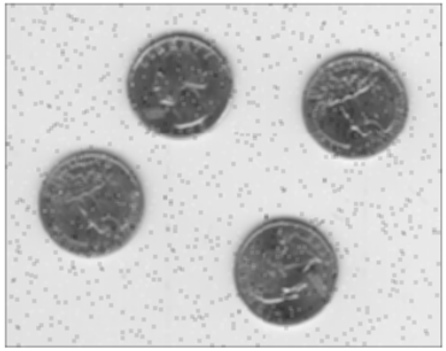


影像強化Image Enhancement

1. Image Enhancement

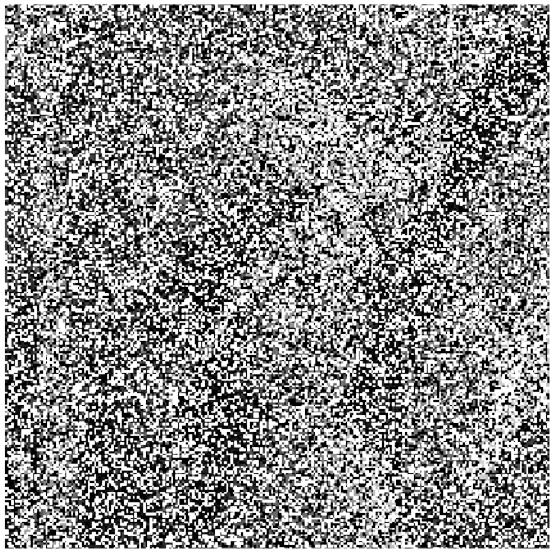
a. Intensity Adjustment

b. Denoise



c. Deblur

除雜訊
Denoising



Chan, Ho, Nikolova L1

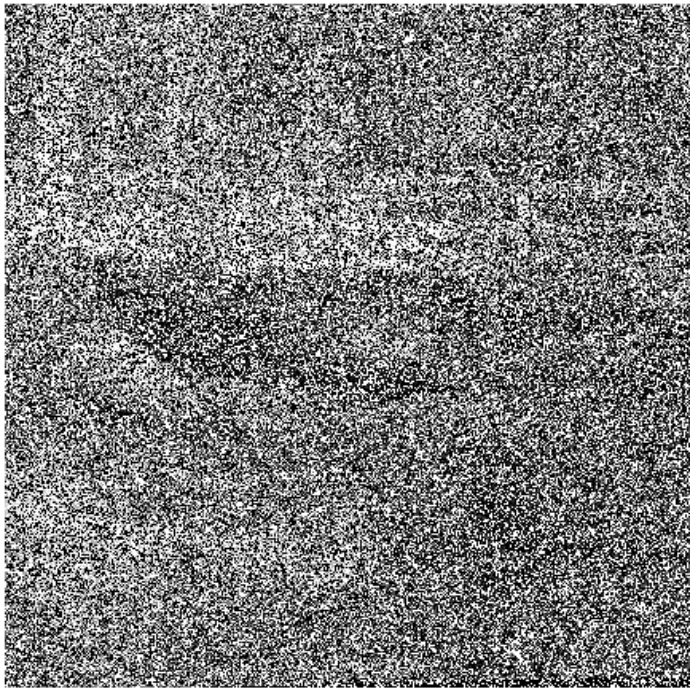
70% Salt-and-Pepper Noise

impulse denoise

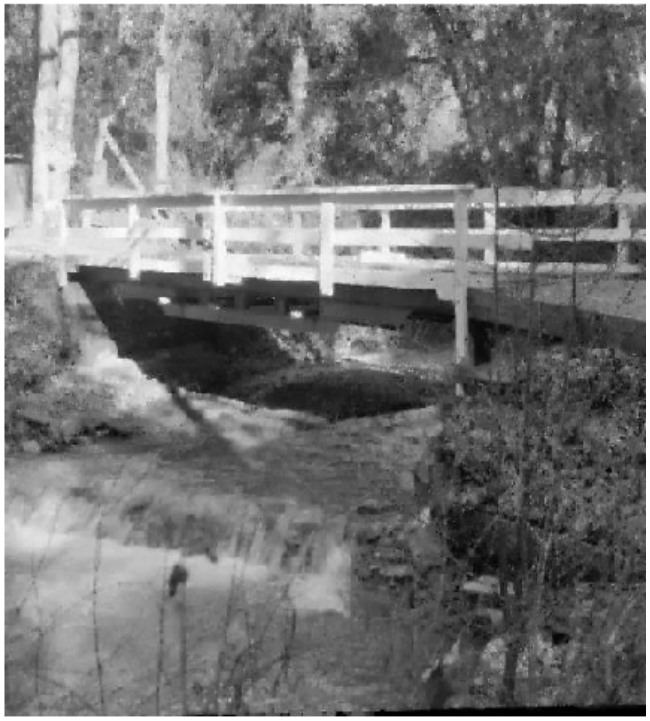
- Impulse noise is Laplace noise. Fidelity function is $\|u - z\|_{L^1}$.
- Edge preserving prior $J(u) = \int |\nabla u|$.
- Denosing by $\min_u F(u)$.

$$F(u) = \beta J(u) + \|u - z\|_{L^1}.$$

除雜訊Denoising

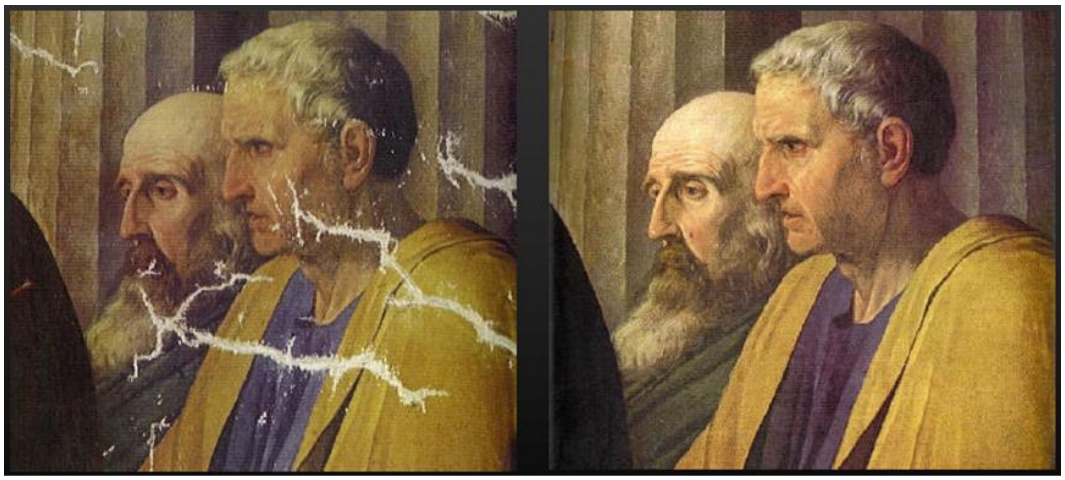


70% Salt-and-Pepper Noise



Chan, Ho, Nikolova

影像填補Image Inpainting



"Image Inpainting : An Overview", Guillermo Sapiro

"Fast Digital Image Inpainting", Manuel M. Oliveira, Brian Bowen, Richard McKenna and Yu-Sung Chiu, Yen Kao

影像填補

- z: observed image, D missing inpainting domain
- Energy functional:

$$E(u, D) = \gamma E_p + \int_{\Omega \setminus D} |u - z|^2$$

- Prior functional:

TV: $E_p = \int_\Omega |\nabla u|$

Elastica: $E_p = \int_\Omega \phi(\kappa) |\nabla u|$

where

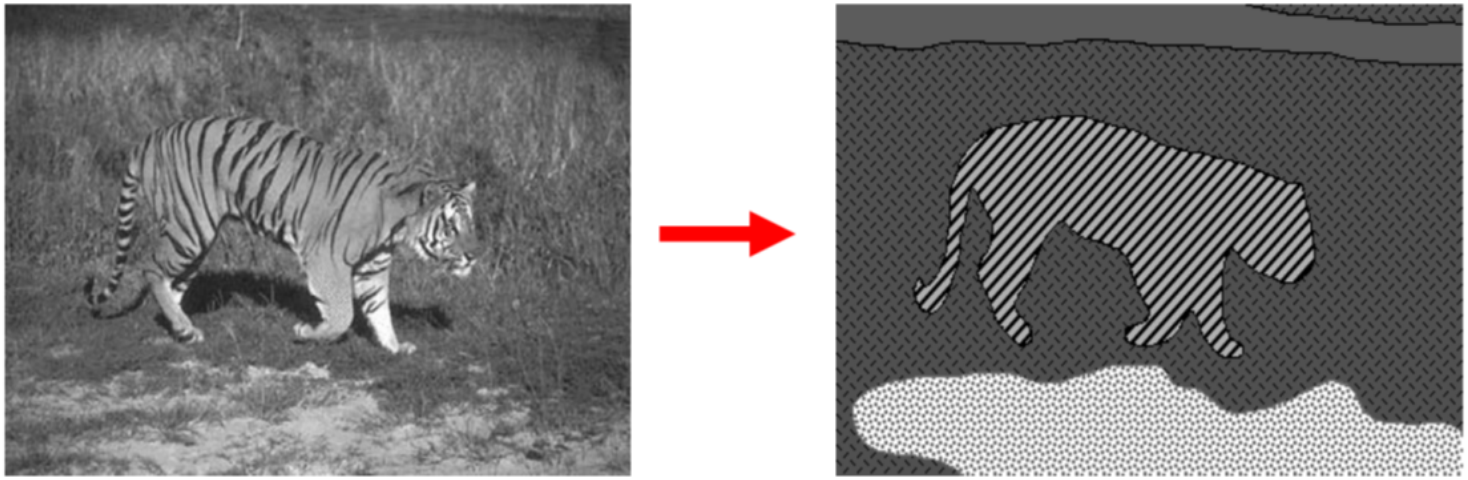
$$\phi(s) = \alpha + \beta s^2, \ \kappa = \nabla \cdot \left[\frac{\nabla u}{|\nabla u|} \right].$$

影像分割Image Segmentation

What is Image Segmentation?

- **Identify objects from background**
- **Partition an image domain into disjoint regions of interest**

$$X = \bigcup_{i=1}^{N} R_i,\ R_i \cap R_j = 0 \ \ for\ i \neq j$$



Segmentation Principles: discontinuity and similarity

•Chiu-Yen Kao

影像對位Image Registration

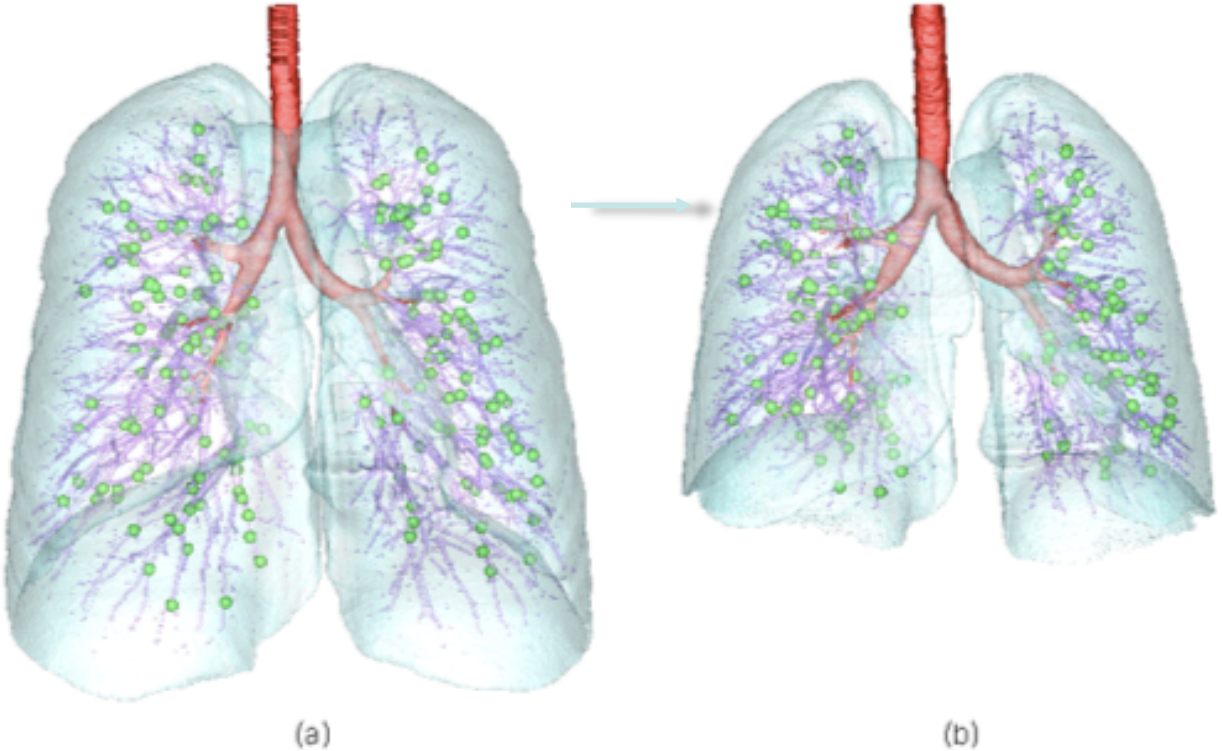
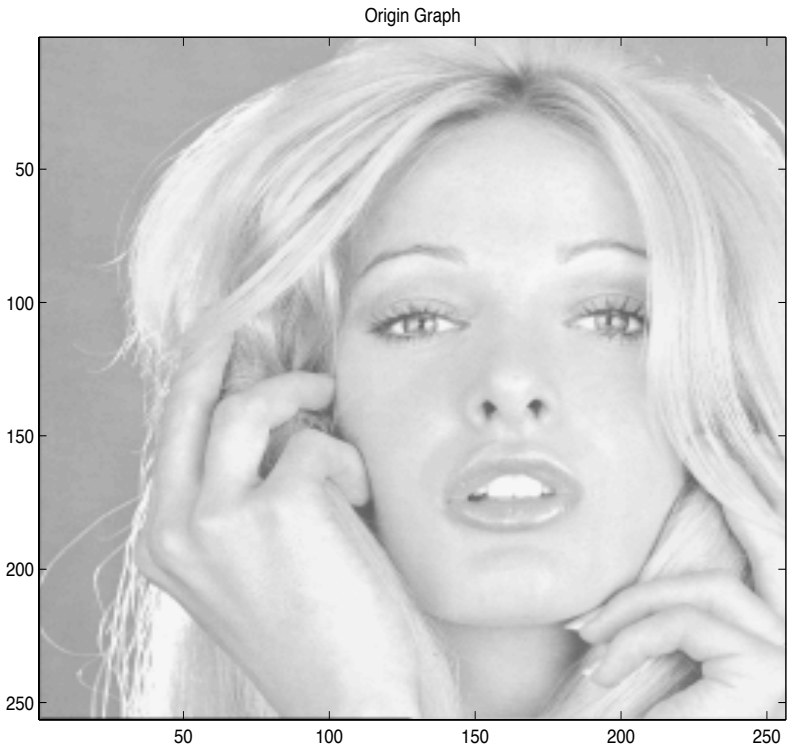


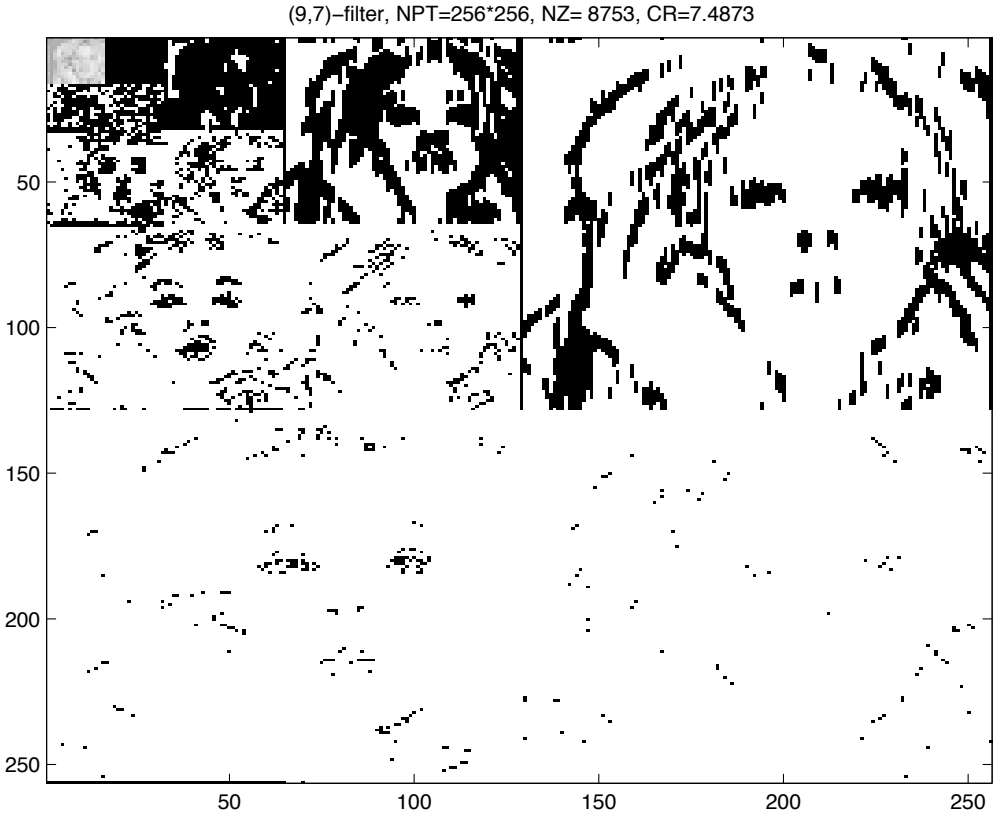
FIG. 4.1. *3D CT images of lungs. Branch points are marked by green dots. (a) is at TLC (total lung capacity), (b) at FRC (functional residual capacity). The lungs, the airway tree, and the vessel tree are marked by cyan, red, and purple, respectively*

P Chen, CL Lin , IL Chern, A PERFECT MATCH CONDITION FOR POINT-SET MATCHING PROBLEMS USING THE OPTIMAL MASS TRANSPORT APPROACH, SIAM J. Image Sciences 2013.

影像表現Image Representation

- Multi-scale representation (wavelets)





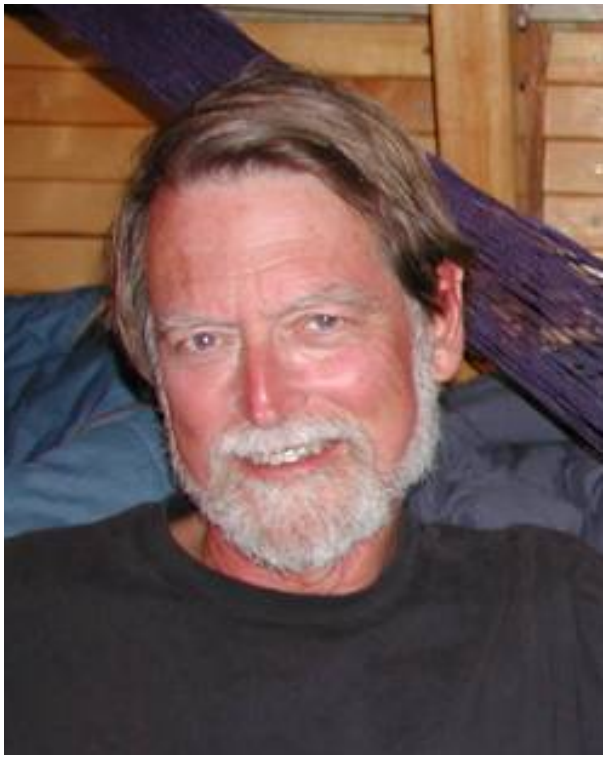
影像壓縮Image Compression

1:1 23:1
144:1

JPEG: Discrete Cosine Transform (Wiki)

Stanley Osher

Tony F Chan



David Mumford

Some Applied Mathematicians in Image Processing

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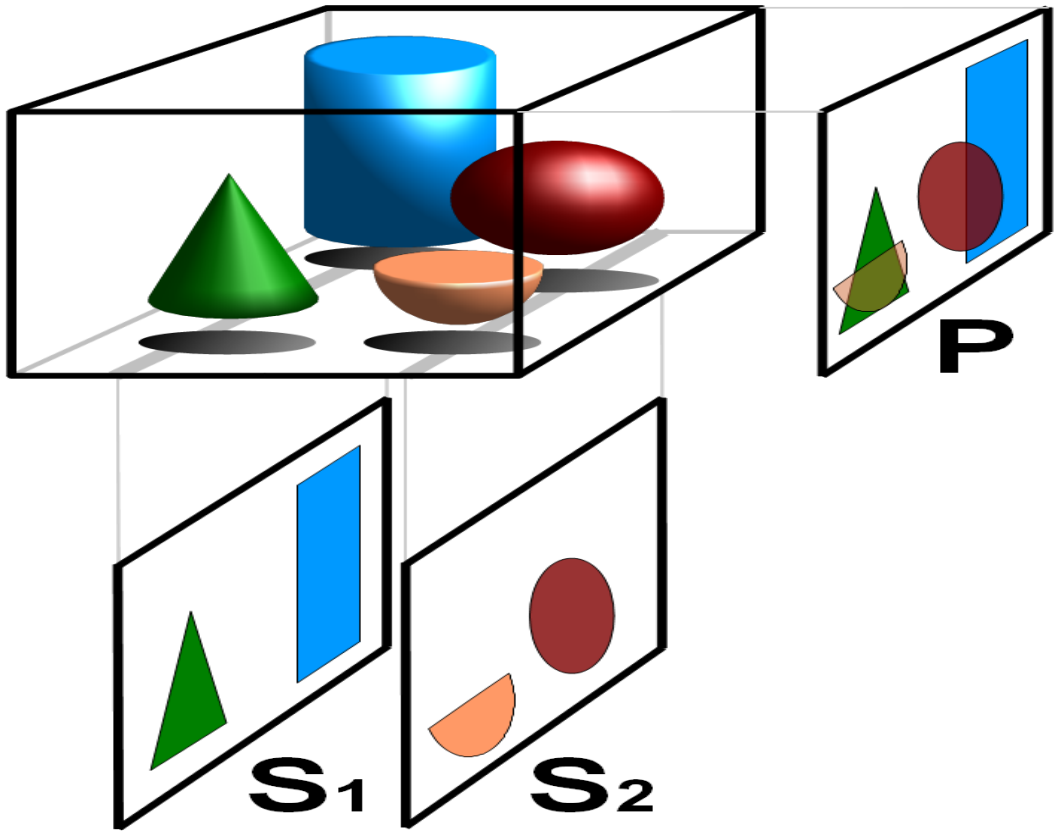
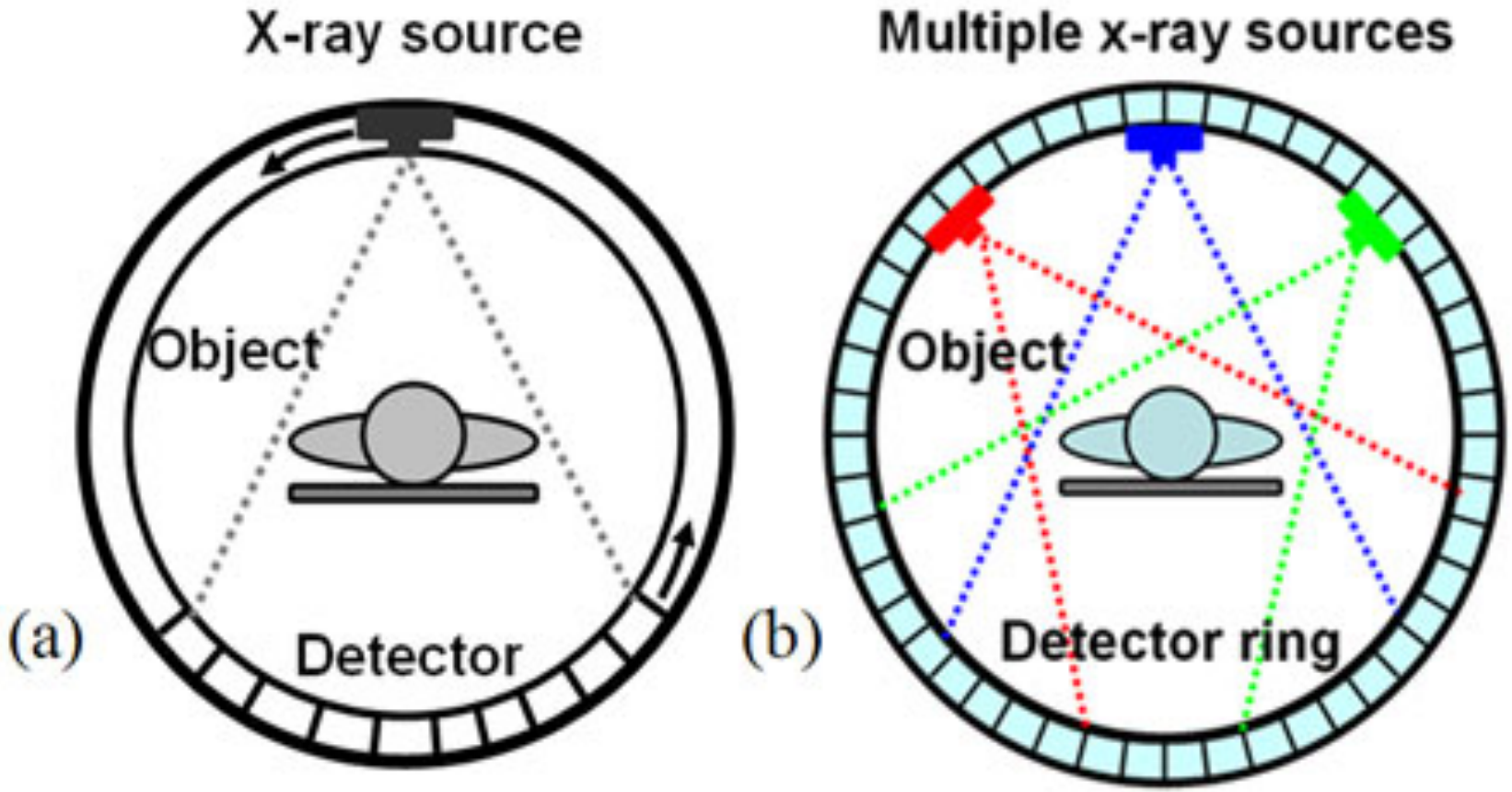
- How Snow is simulated in the movie FROZEN

斷層攝影Tomography



Basic principle of tomography: superposition free tomographic cross sections S1 and S2
compared with the projected image P

X光電腦斷層攝影X-ray CT



X-ray Computed Tomograph

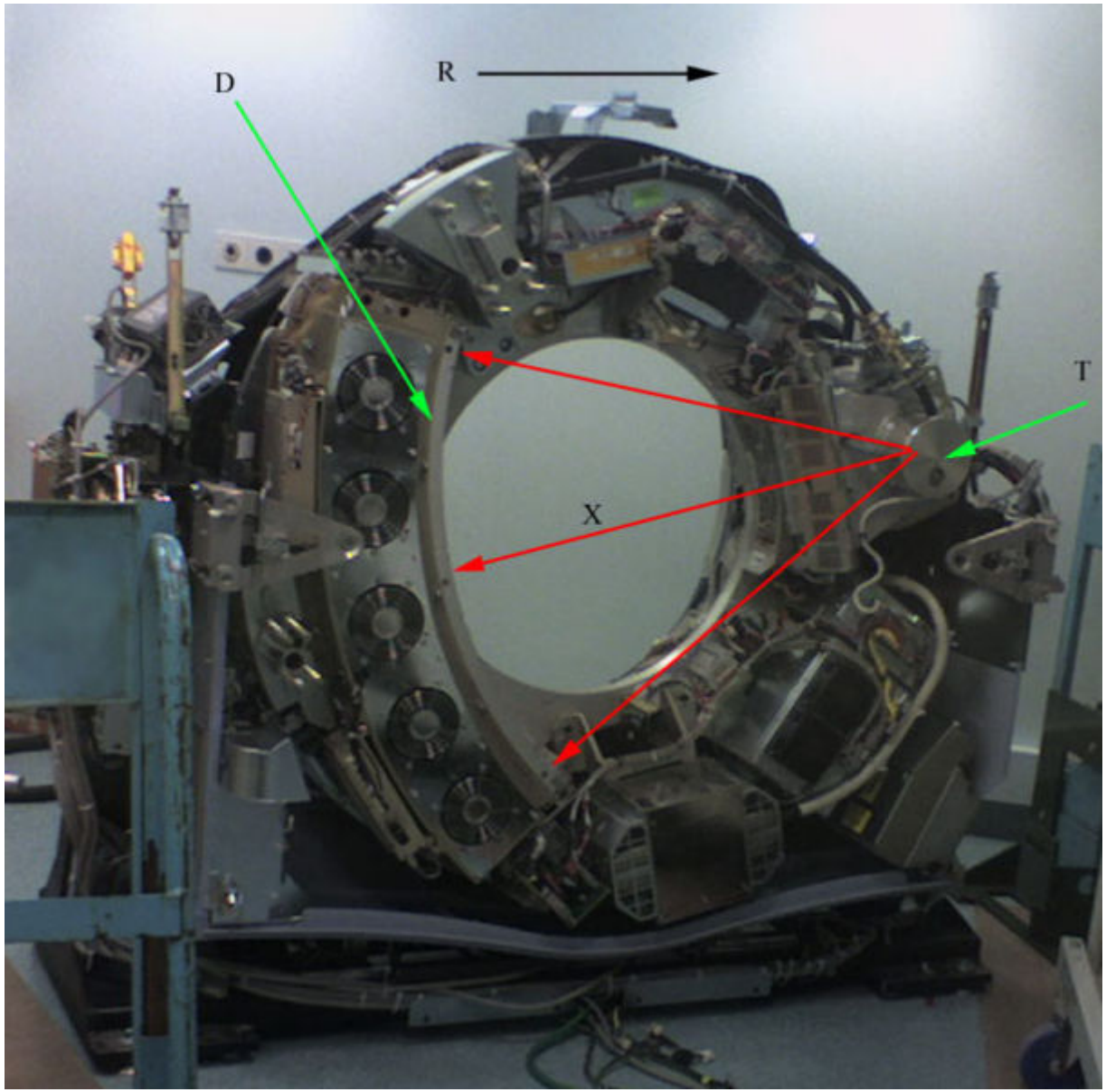


Image Reconstruction

- Tomographic reconstruction :
  - Radon transform

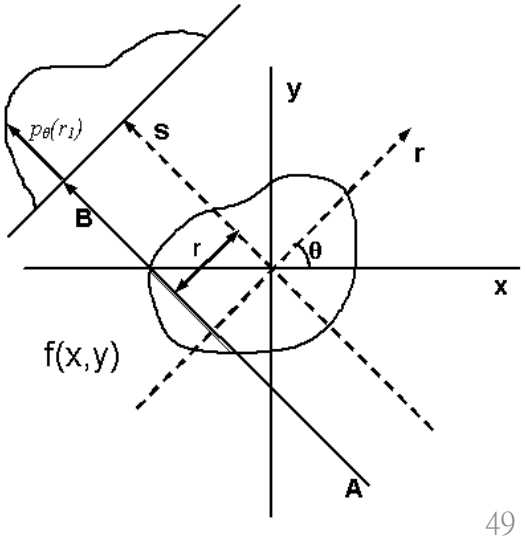
$$Rf(\theta, r) = \int_{x \cdot \theta = r} f(x)dx, \ \theta \in S^1$$

  - Imaging model

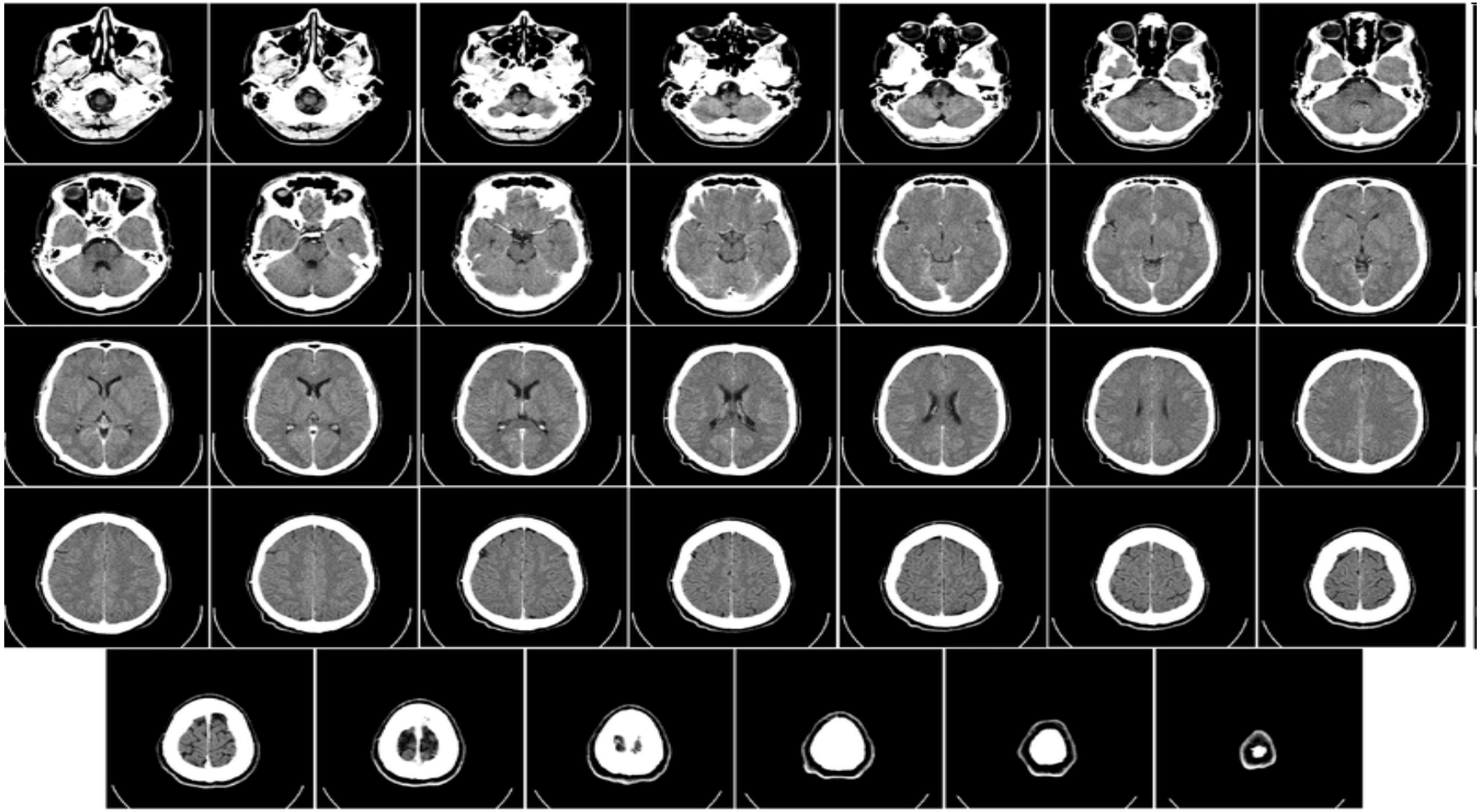
$$z = Ru + n$$

  - Image reconstruction

Given z, reconstruct u



Reconstructed images by CT



Nobel winners for CT (1979)

Godfrey Hounsfield

Allan McLeod Cormack

現代生活中的數學

- How google works
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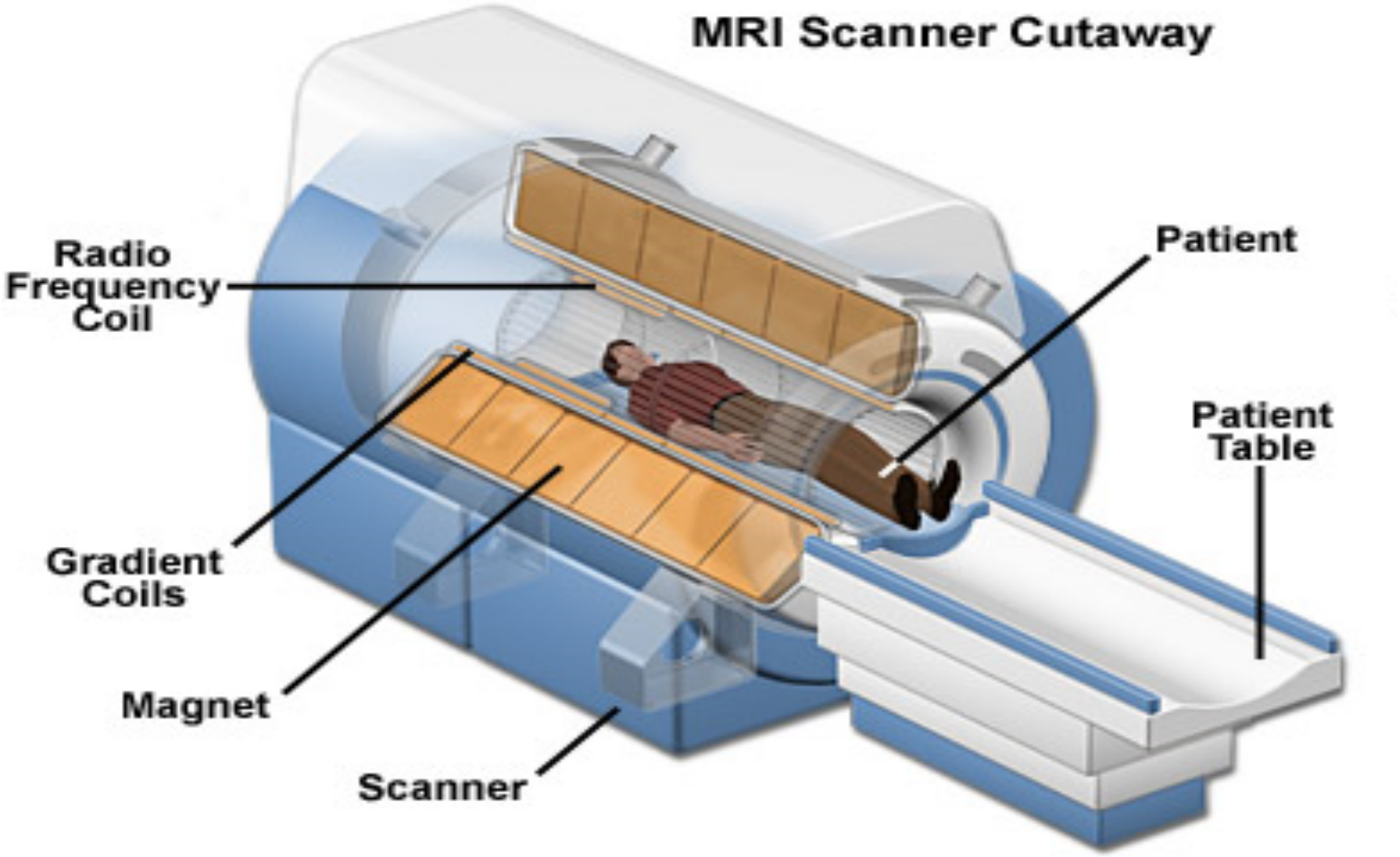
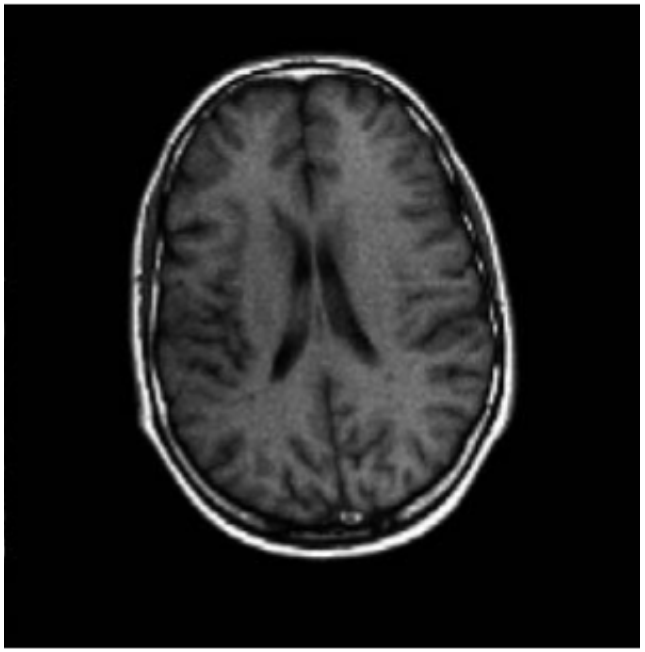
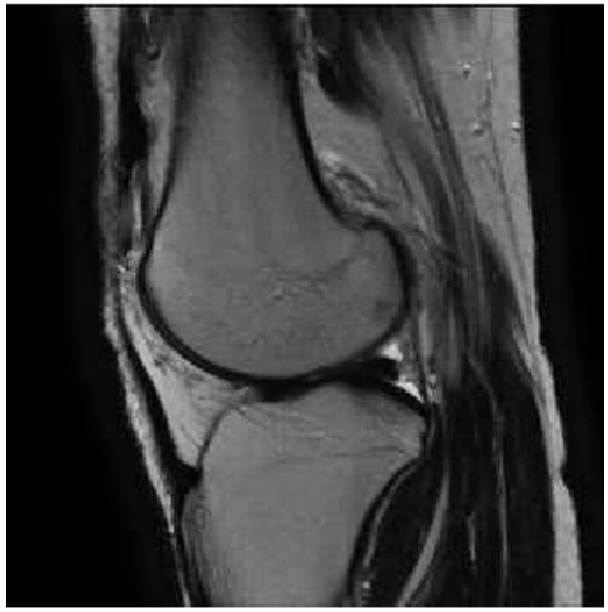
Magnetic Resonance Imaging (MRI)



MRI images





Typical resolution: 256 x 256 or 512 x 512

MRI history

The Nobel Prize in Physics 1952

"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"

Felix Bloch

◑ 1/2 of the prize

USA

Stanford University
Stanford, CA, USA

Edward Mills Purcell

◑ 1/2 of the prize

USA

Harvard University
Cambridge, MA, USA

The Nobel Prize in Physiology or Medicine 2003

"for their discoveries concerning magnetic resonance imaging"

Paul C. Lauterbur

◑ 1/2 of the prize

USA

University of Illinois
Urbana, IL, USA

Sir Peter Mansfield

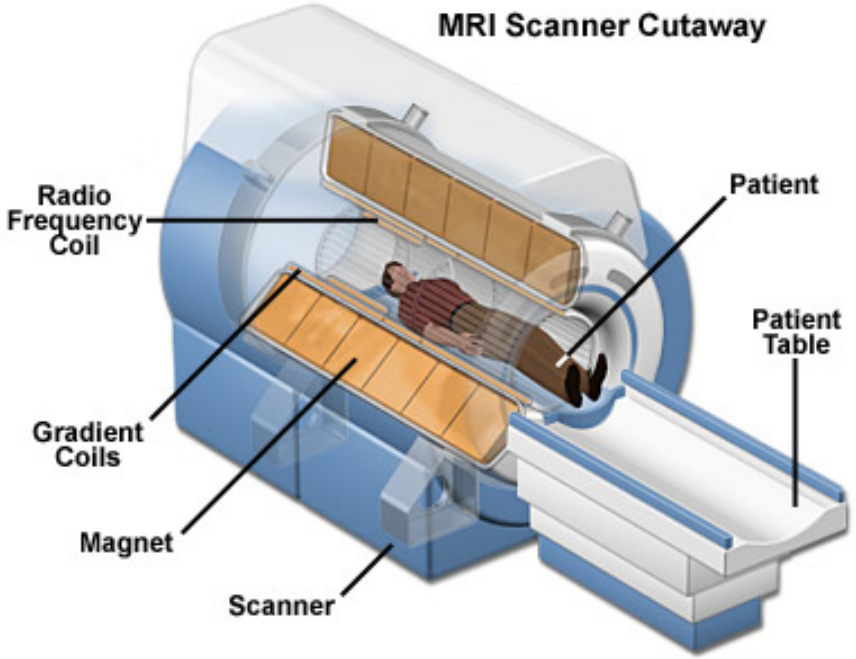
◑ 1/2 of the prize

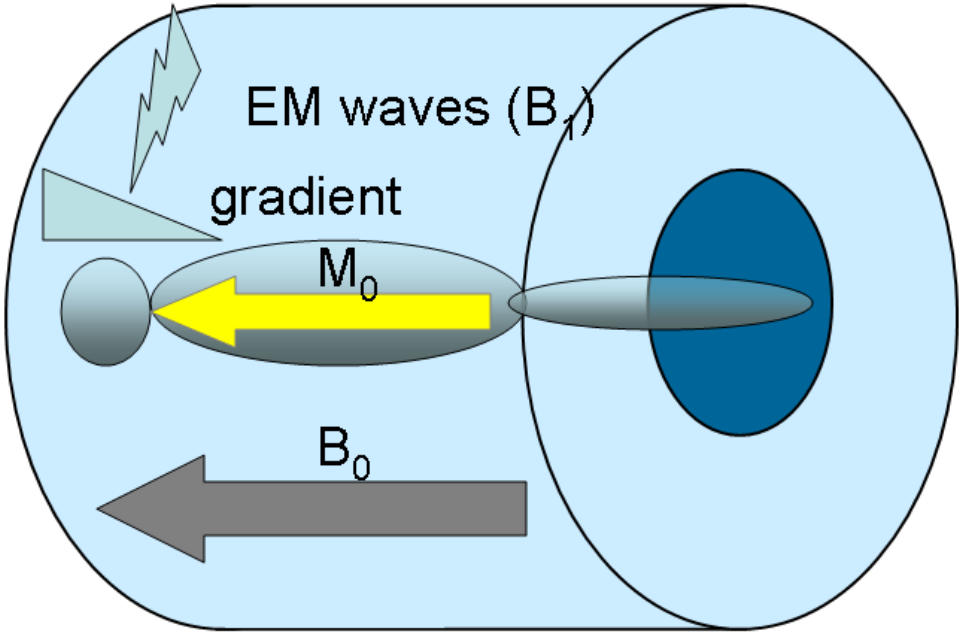
United Kingdom

University of Nottingham,
School of Physics and Astronomy
Nottingham, United Kingdom

核磁共振
用磁場操控

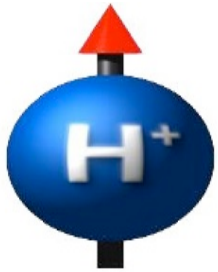
- 鬆弛: 主磁場 B0
- 激發: 無線電波場(RF) B1
- 編碼: 梯度磁場 G

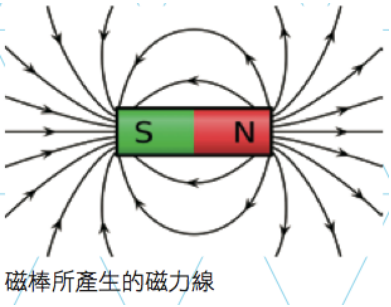




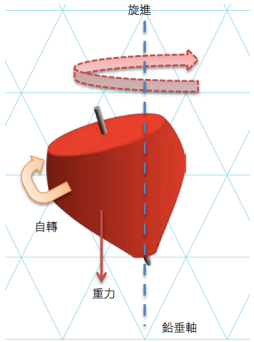
核磁共振的基本原理 I

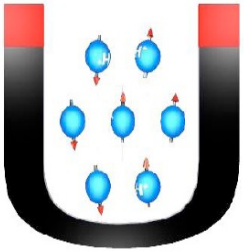
- 一些原子核具有核自旋，就像陀螺一般
- 具核自旋的原子核會形成「磁偶極矩」,就像一個小磁鐵般,在磁場中進行旋進，類比於陀螺在重力場下會進行旋進
- 旋進頻率稱作拉姆頻率，等比於磁場強度。
- 當核偶極矩的方向不平行於磁場時,就會像旋轉的小磁般,造成空間 磁通量(通過磁場時的大小度量)的變化,可藉由感應線圈,偵測得到核旋進的訊號。
- 醫學用的核磁共振儀,主要就是偵測我們體內氫原子核的自旋





磁棒所產生的磁力線

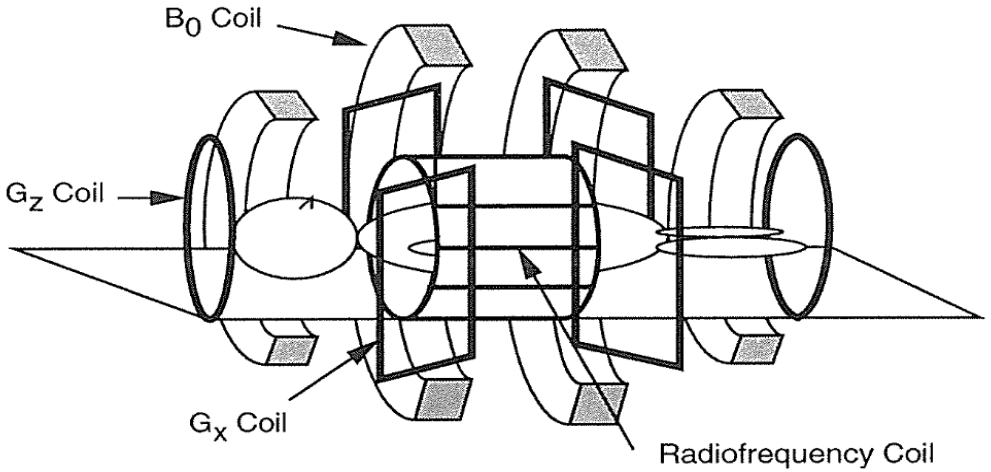




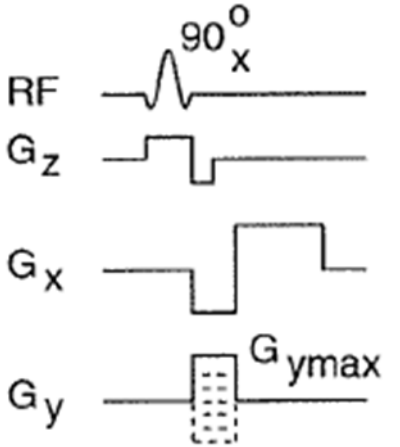
核磁共振的基本原理 II

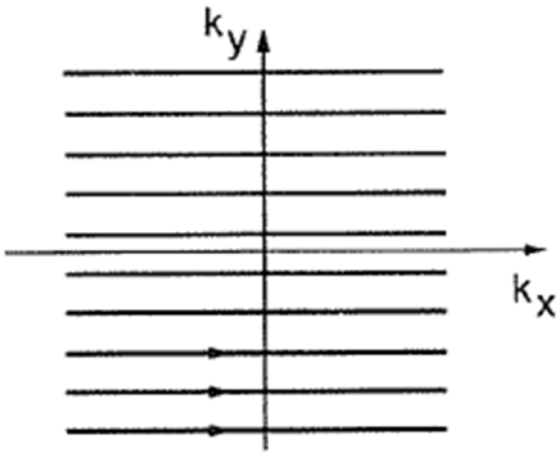
- 激發: 將磁化向量轉到橫向的過程
- 鬆弛: 磁化向量恢復到主磁場方向的過程
- 編碼: 在每次的鬆 弛過程中,打開不同型態的梯度 磁場,便可以操控磁化向量在不同位置的旋進頻率,這個過程 稱 作「編 碼」
- 由這些編碼所得的訊號,我們可以反推空間中磁化向量強度的分布
- 在腦中,不同介質(骨骼、 灰質、白質)的磁化強度不同, 因此磁化向量強度的分布,即反 映了腦中的結構。

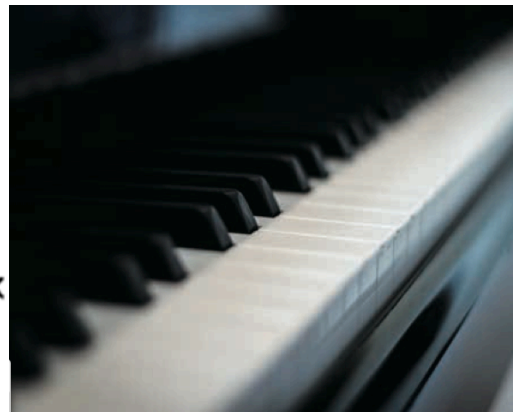
核磁共振編碼: 就像鋼琴演奏一般



$$B(t) = B_0 + B_1(t) + G(t)$$







快速核磁共振造影

- 醫學造影通常希望
  - 低劑量
  - 高精度
  - 快速造影
- 最近的突破: 壓縮感知(Compressive Sensing)

壓縮感知

- In MRI, we want to reconstruct an $N \times N$ image f from $\hat{f}(\omega)$ defined by

$$\hat{f}(\omega) = \sum_{t \in \mathbb{Z}_N^2} f(t) e^{-2\pi i \omega \cdot t/N}$$

  with $\omega \in \Omega$ with $\#\Omega << N^2$.

- Suppose $N = 512$. We choose Ω consisting 22 radial lines with 512 uniform sample points on each line.
- The reconstruction can be exact if f is sparse.

E. Candes, J. Romberg, T. Tao 2006
David Donoho 2006

Compressive sensing

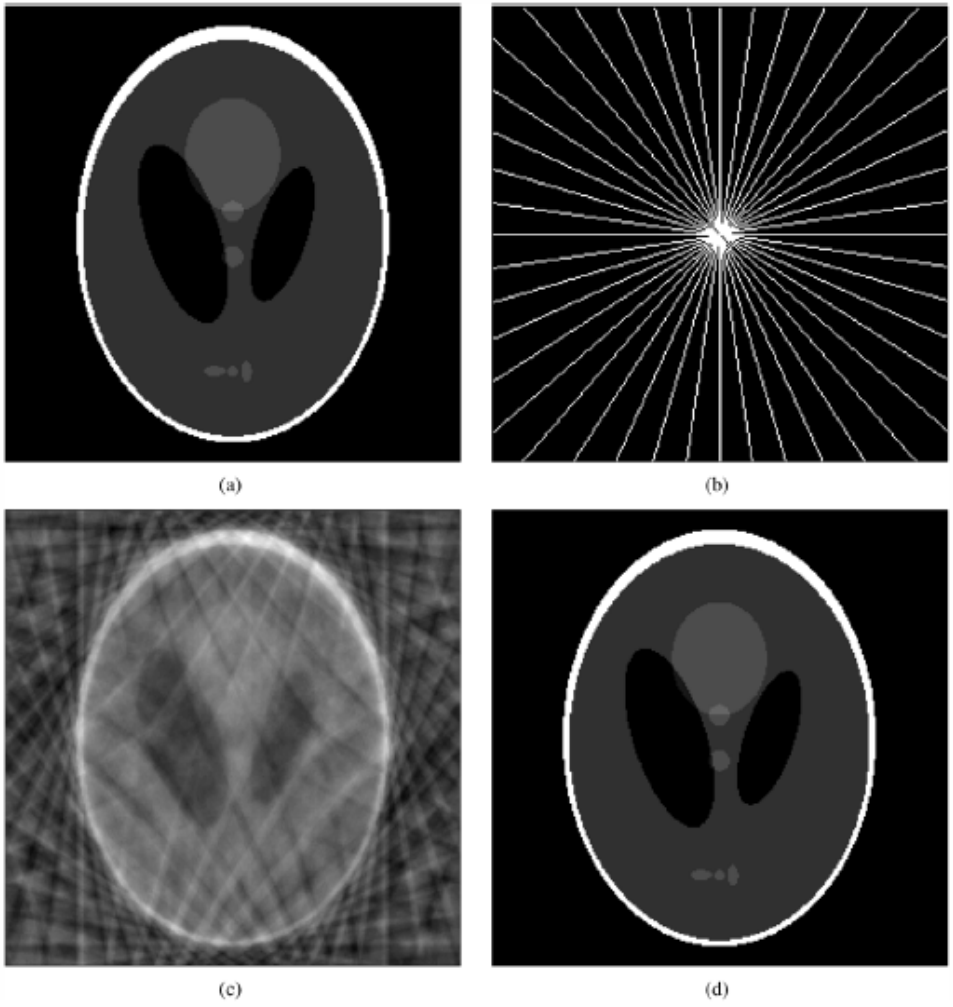


Fig. 1. Example of a simple recovery problem. (a) The Logan–Shepp phantom test image. (b) Sampling domain Ω in the frequency plane; Fourier coefficients are sampled along 22 approximately radial lines. (c) Minimum energy reconstruction obtained by setting unobserved Fourier coefficients to zero. (d) Reconstruction obtained by minimizing the total variation, as in (1.1). The reconstruction is an exact replica of the image in (a).

Candes,Romberg,Tao 2006

Compressive sensing in Fourier space

- The result of filtered backprojection is poor if Ω is sparse.
- The result of TV regularization is exact if f is sparse:

$$\min_u \|\nabla u\|_1 \text{ subject to } \hat{u}(\omega) = \hat{f}(\omega), \omega \in \Omega.$$

Compressive Sensing (2004)

David Donoho

Michael Lustig

Emmanual Candes

Terence Chi-Shen Tao

Candes, Romberg, Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 52, NO. 2, FEBRUARY 2006 cited 4894

DL Donoho, Compressed sensing, Information Theory, IEEE Transactions on, 2006 , cited 6716

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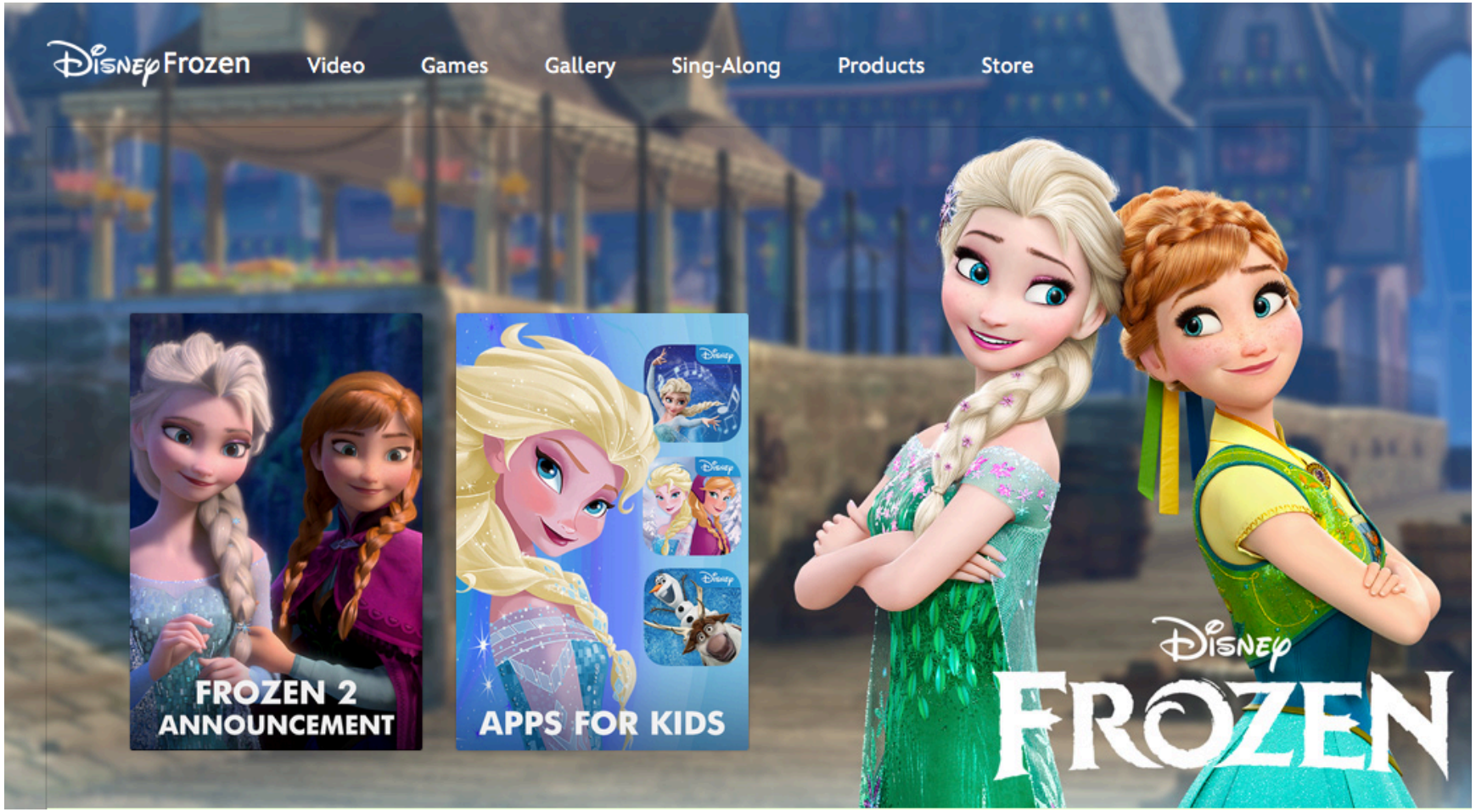
迪士尼的冰雪奇緣



YouTube: Frozen

冰雪奇緣的雪是怎麼模擬的

- 雪是一位UCLA數學系的教授Joseph Teran團隊所做的電腦動畫
- 所用的方法是以數值方法解彈塑性力學方程式。

$$\frac{D\rho}{Dt} = 0, \quad \rho\frac{D\boldsymbol{v}}{Dt} = \nabla\cdot\boldsymbol{\sigma} + \rho\boldsymbol{g}, \quad \boldsymbol{\sigma} = \frac{1}{J}\frac{\partial\Psi}{\partial\boldsymbol{F}_E}\boldsymbol{F}_E^T,$$

A material point method for snow simulation

Alexey Stomakhin*† Craig Schroeder† Lawrence Chai* Joseph Teran*† Andrew Selle*

†University of California Los Angeles *Walt Disney Animation Studios

Abstract

Snow is a challenging natural phenomenon to visually simulate. While the graphics community has previously considered accumulation and rendering of snow, animation of snow dynamics has not been fully addressed. Additionally, existing techniques for solids and fluids have difficulty producing convincing snow results. Specifically, *wet* or *dense* snow that has both solid- and fluid-like properties is difficult to handle. Consequently, this paper presents a novel snow simulation method utilizing a user-controllable elasto-plastic constitutive model integrated with a hybrid Eulerian/Lagrangian Material Point Method. The method is continuum based and its hybrid nature allows us to use a regular Cartesian grid to automate treatment of self-collision and fracture. It also naturally allows us to derive a grid-based semi-implicit integration scheme that has conditioning independent of the number of Lagrangian particles. We demonstrate the power of our method with a variety of snow phenomena including complex character interactions.

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation I.6.8 [Simulation and Modeling]: Types of Simulation—Animation

Keywords: material point, snow simulation, physically-based modeling

Links: DL PDF WEB

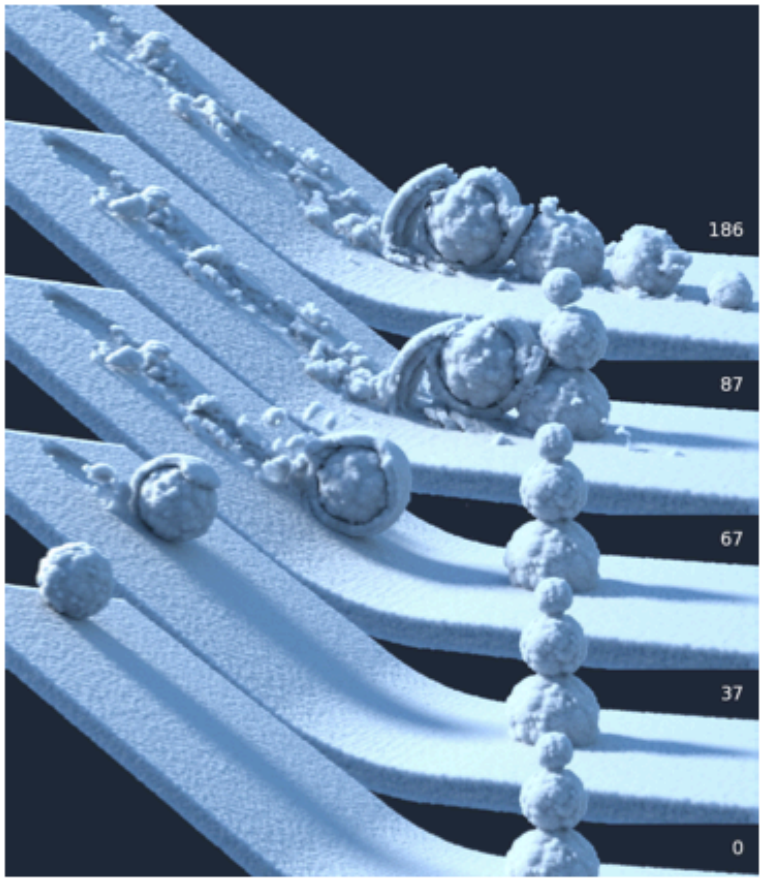
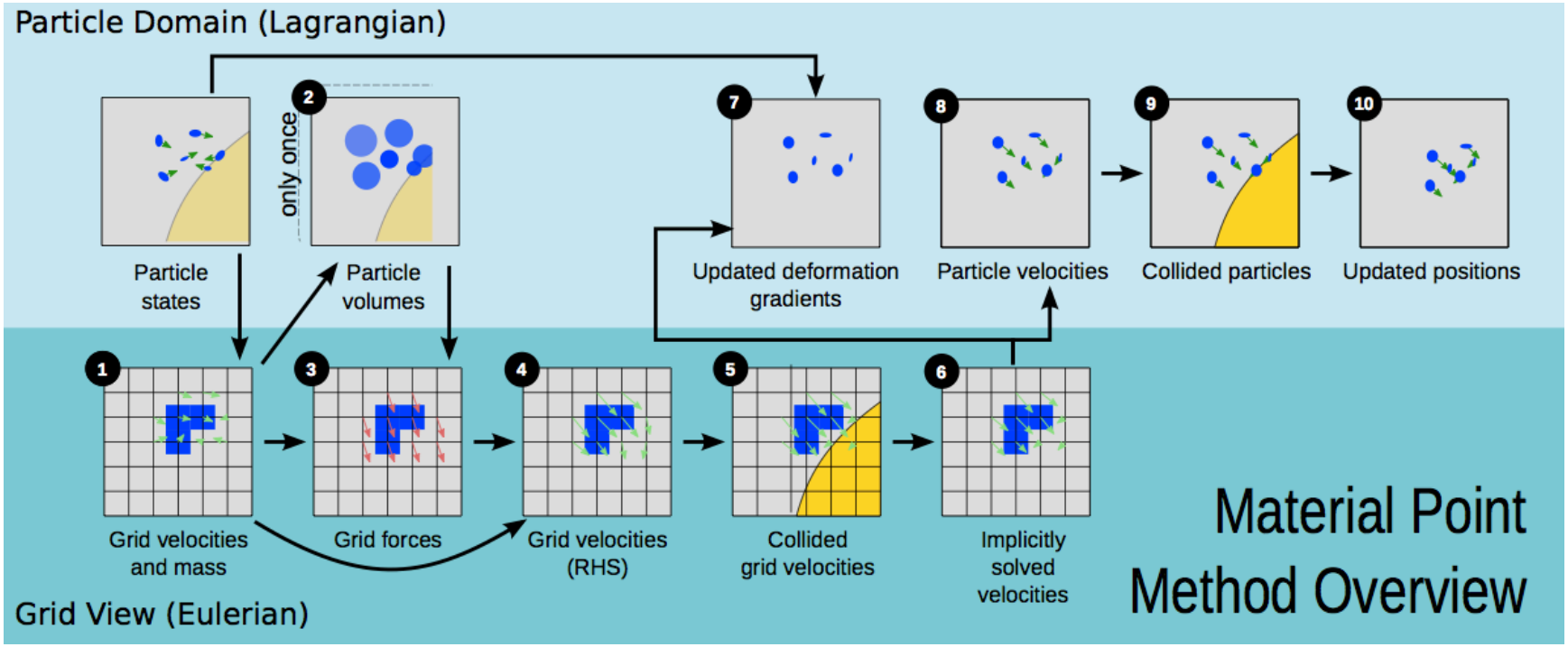
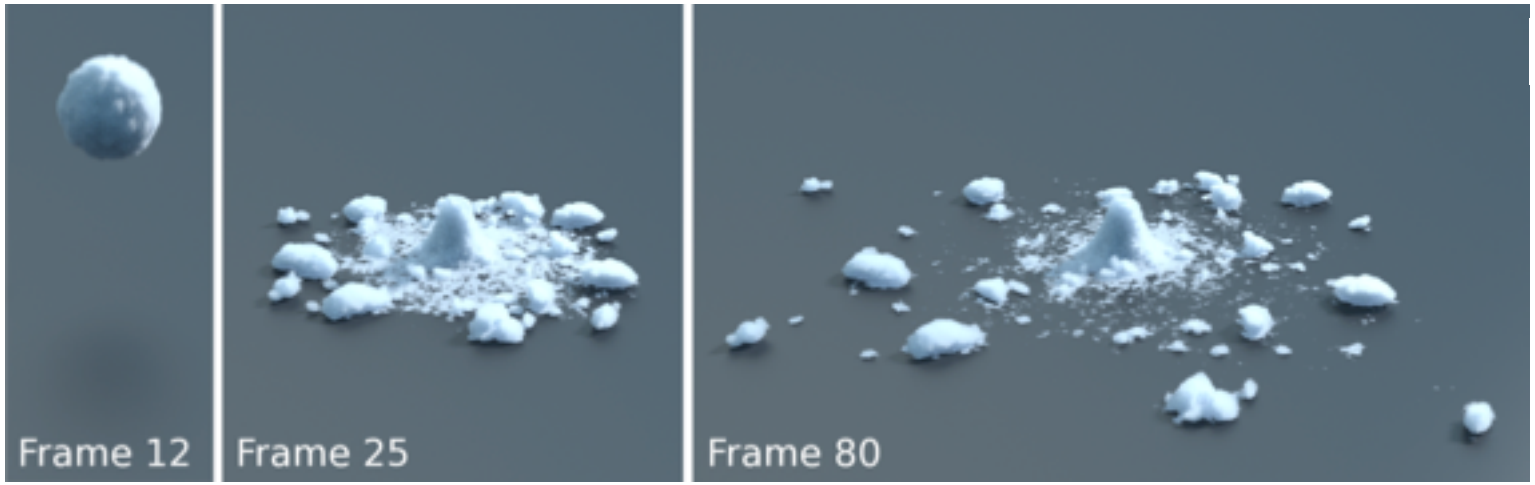


Figure 1: Rolling snowball. *As the snowball moves down the hill, compressed snow sticks, demonstrating that we can handle so-called packing snow effect.* ©*Disney.*

1 Introduction





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[Disney's Frozen - A Material Point Method For Snow Simulation]

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- 活用的數學，大幅改進了我們的生活。

億

新時代 新思維 新數學

- 傳統數學在新時代要有新思維
- 新的應用課題賦予傳統數學新的生命
  - 線性代數中在數據分析、影像處理十分重要
  - 各種大尺度數值計算十分重要，在電腦動畫也有新的應用。
  - 優化、數學規劃等都有新的應用
  - 圖論、離散幾何在大數據、社群網路研究等有新的應用
  - 微分方程、偏微分方程、變分學、反問題等仍是連續世界建模的基本工具
  - 機率、統計、不確定性的量化分析等也是建模的工具

做中學

- 多接觸應用問題
- 從實作中學
- 從簡單的東西做起
- 學習將複雜的東西化成簡單的東西

一個小故事

- 一個數學系畢業生組織的讀書會
  - Taiwan R User group and Data Mining
  - http://www.meetup.com/Taiwan-R/
  - Python, Git
- R 是目前最紅的 Open Source 統計語言，而且，不只是對於傳統的統計檢定來說，或是，對於新世代的 Machine Learning 和 Data Mining 的許多技術而言，R 都有很完整的套件支援。因此，可以很快速的在R上面運用各種 Machine Learning 的技術來分析資料。
- 他們開了一家Data Mining 公司。
- 大量數據分析: http://www.revolutionanalytics.com/

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謝謝大家